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SENSITIVE MOVING COIL GALVANOMETERS.

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THE following article deals primarily with the principles underlying the construction of sensitive moving coil galvanometers and with various experimental devices for increasing their sensibility. Under this latter head are included tremor-shielding devices. Incidentally, by treating of the galvanometer as a motor some of its fundamental properties are conveniently exhibited; the results obtained lead naturally to a brief discussion of the relative sensibility of different types of instrument.

1. Fundamental Formulæ.

The essentials of the mathematical theory of the moving coil galvanometer are comparatively well known.[1] It therefore seems sufficient here to merely state as much of the theory as will be needed in the present discussion.

In any galvanometer whatever:

Let K be the moment of inertia.

Q, the moment of torsion of the controlling force.

I, the current.

ϑ, the angular deflection.

R, the resistance of the whole circuit.

G, the dynamic constant, that is, the deflecting torque produced

[1] F. Kohlrausch, Lehrbuch der praktischen Physik, 9th Ed, p. 450; Th. Des Coudres, Zeitschrift fur Electrochemie, 3, 1896–1897. O. M. Stewart, Phys. Rev., 16, p. 158, 1903. W. Jaeger, Zeitsch. f. Instrumentenkunde, 23, pp. 261, 353, 1903.

by unit current, this depends on the strength of the magnets and the arrangement of the coils.

$T/2$ the undamped half period.

Then

$$\frac{T}{2} = \pi \sqrt{\frac{K}{Q}} \tag{1}$$

The sensibility,

$$\frac{\theta}{I} = \frac{G}{Q}. \tag{2}$$

The counter electromotive force developed in the coil by its own motion is

$$G\frac{d\theta}{dt}$$

the corresponding current is:

$$\frac{G\,d\theta}{R\,dt}$$

the damping effect of this current is a torque,

$$\frac{G^2}{R}\frac{d\theta}{dt}, \quad = D'\frac{d\theta}{dt}$$

whence D', a damping coefficient, $= G^2/R$. (3)

It will presently appear that the damping due to this cause involves no loss of energy during deflection; it is, in a somewhat special sense, conservative damping.

For the moving coil galvanometer in particular:

Let r be half the width of the coil.

l, the total length of vertical wire in the coil.

H the field strength.

For simplicity the mirror and the "dead horizontal wire at the ends of the coil may be neglected, since these can easily be brought in as corrections to the simplified formulæ.

Then

$$K = lr^2 \times (\text{mass of unit length of wire}), \tag{4}$$

$$G = Hlr. \tag{5}$$

The motion of a galvanometer coil or magnet is, more or less exactly, damped harmonic of a familiar type. It is well known

that such motion dies down most rapidly when the damping is critical — that is, just barely aperiodic, and the condition for this critical damping is [1]

$$D = 2\sqrt{QK}, \tag{6}$$

and the moving coil galvanometer is actually found to work best when so damped. Moreover, for maximum sensibility it is evident that no energy should be wasted, hence the damping should be all conservative, and D may be taken equal to D'. This condition can be approximately realized with the moving coil galvanometer.[2] Then, assuming critical damping, we have, from equations (3), (5), (6),

$$G^2 = H^2 l^2 r^2 = 2R\sqrt{QK}, \tag{7}$$

$$H^2 = \frac{2R\sqrt{QK}}{l^2 r^2}. \tag{8}$$

The expression for the sensibility, (2), can now be transformed by eliminating G and inserting T by (1), (7)

$$\frac{\theta}{I} = \sqrt{\frac{2R\,T/2}{\pi Q}}, \tag{9}$$

and similarly,

$$\frac{\theta}{I} = \sqrt{\frac{2R\,(T/2)^3}{\pi^3 K}}. \tag{10}$$

These formulæ can be expressed in more convenient units. If R is in ohms, I in microamperes, θ in two-thousandths of a radian — that is, millimeters at a meter distance, allowing for the doubling effect of the mirror — then the sensibility in megohms — which is numerically the same as millimeters per microampere at one meter — becomes:

$$S = 5.02\sqrt{\frac{R\,T/2}{Q}}, \tag{11}$$

or

$$S = 1.60\sqrt{\frac{R\,(T/2)^3}{K}}. \tag{12}$$

[1] O. M. Stewart, loc. cit.

[2] W. Jaeger, op. cit., p. 355.

A corresponding expression can be found for H by eliminating T from (8) and changing units. For critical damping,

$$H = \frac{25{,}200\sqrt{RQ\frac{T}{2}}}{lr}. \tag{13}$$

The above formulæ are sufficient for the designing of moving coil galvanometers, given the properties of the magnets and the wire. Another relation will prove convenient a little later. From equation (7) it appears that in *any* galvanometer if T is increased by diminishing Q, the value of G required for critical damping becomes less.

$$G \propto \frac{1}{\sqrt{T}}. \tag{14}$$

That is, it is easier to get critical damping as T increases.

For the moving coil galvanometer the required value of H diminishes also, according to (8), if T is increased by increasing l. In this case

$$H \propto T^{-\frac{1}{2}}. \tag{15}$$

2. The Galvanometer as a Motor.

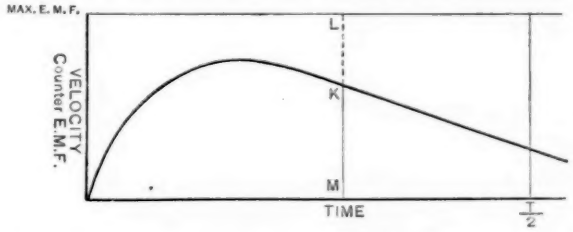
One great advantage of the moving coil galvanometer is the ease with which large values of G are obtainable. These at first sight promise great sensibility, and suggest the use of powerful electromagnets to secure it. It is now well known, however, that the possibilities in this direction are limited by the overdamping which results when the field strength exceeds the critical value. This damping is usually represented as the result of a current generated by the motion of the coil, and opposite to the current which is being measured. In reality, of course, there can not be two currents in the same circuit, and it is simpler to regard the damping during deflection as due to a counter electromotive force which diminishes the driving current. A galvanometer is a motor, and overdamping is simply the inevitable result of an attempt, in trying to turn a heavy coil too far in a given time, to get more energy out of the current source than it is capable of giving.

Some notion of the magnitude of the counter E.M.F. in a critically damped galvanometer may be obtained from Fig. 1. The upper

curve represents the variation in the angular velocity, as computed from the equation of motion, during the time of a free half swing. Since the counter E.M.F. is proportional to the velocity it also is represented by this same curve if a suitable scale be taken. If, now, the upper horizontal line shows to the same scale the value of the driving E.M.F., the dotted ordinates, measured downward from this line, represent the resultant E.M.F., and are therefore proportional

Fig. 1a.

Velocity-time curve for a critically damped galvanometer, which is also counter-electromotive-force-time curve. The dotted ordinates are proportional to current.



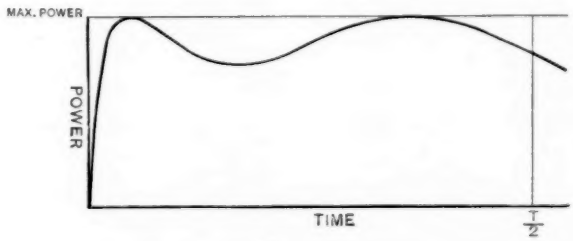


Fig. 1b.

Power-time curve. The area under the curve is the current energy utilized in the first half-period; the area of the parallelogram is the maximum available energy.

to the actual current. It may be noted that this falls at one time to about one fourth its maximum value. Since the upper and lower ordinates at any point, as LK, MK, are proportional to current and counter E.M.F. respectively, their product will be proportional to the power expended on the coil. Values of this product are shown in the power curve, and the upper line shows to the same scale the maximum possible power obtainable from the current, or $C^2R/4$. The area under the curve corresponds to the total work done on the coil during the time $T/2$, and this evidently bears a very high ratio to the maximum possible work.[1]

[1] The exact ratio, readily obtainable by analytic methods, is 87.4 per cent.

The above curves represent any galvanometer which has critical, conservative damping, hence all such galvanometers utilize during corresponding periods the same proportion of the energy obtainable from the current. This conclusion, like the curve itself, follows from the equations of motion of the galvanometer, and can be shown to be involved in equation (9). Similarly it can be shown that the proportion of the energy utilized is a maximum for the critically damped galvanometer. This galvanometer is thus somewhat analogous to the Carnot engine. It is, in a way, an ideal galvanometer.

The notion of an ideal galvanometer is often convenient. For instance, such a one — that is, generally speaking, a good moving coil galvanometer — will be more sensitive than any other galvanometer of the same moment of inertia, period, etc., and will usually prove decidedly superior if any heavy mass, such as a pointer or large mirror, is to be moved. Moreover, the sensibility of a corresponding ideal galvanometer may often be advantageously used to determine the limit of possible sensibility of any other. For example, C. H. Abbott has given very full data in regard to a sixteen-coil instrument in use at the U. S. Astrophysical Observatory in Washington.[1] The weight of the magnet system is 10 mg., its moment of inertia, 6×10^{-6}, the resistance is 1.6 ohms, and the (computed) sensibility at 10 seconds half period is 20,000 megohms. From formula (12) the ideal sensibility is 26,000 megohms. For 1.5 seconds the sensibility is 450, which is now only 30 per cent. of the ideal — a decrease accounted for by (14). A similar 4-coil instrument, too delicate for use under ordinary conditions, whose system weighs 2 mg., gives a sensibility of 96,000 megohms at 10 seconds half period. Its ideal sensibility for 10 seconds is in the neighborhood of 50,000. Thus the instrument at 96,000 megohm sensibility would be overdamped, and really inefficient, unless the total resistance were rather high; and if this were nearly three times that of the galvanometer the damping would be aperiodic from magnetic action alone. We thus demonstrate from energy considerations that for moderately long periods this galvanometer has attained at ordinary cooking conditions the maximum sensitiveness possible without a diminution in the size of the moving mass.

[1] C. H. Abbott, Astrophys. Jour., 18, p. 1, 1903. S. P. Langley, Astroph. Obs. Annals, 1, pp. 59, 244, 1900.

The energy available in a galvanometer is a function of current, total resistance, and time, and these are in one sense quantities independent of the instrument itself, representing the conditions it has to meet. Given these conditions, sensibility in an ideal galvanometer depends on the deflection obtainable with a given amount of energy — that is, upon the smallness of the controlling force and coil — a conclusion also derivable from equations (9) and (12). In galvanometers of the Kelvin type special effort is also needed to increase the value of G, which, as we have just seen, is almost never as great as is desirable. The various improvements in this type of instrument have generally been secured by attention to the quality of the magnets and to the coils as well as by reduction in the size of the moving parts.

3. Comparisons of Sensibility.

If any galvanometer has its period lengthened by a change in Q the sensibility will increase as the square of this change. Moreover, the number of turns in a coil of given size is approximately proportional to the square root of the resistance. Hence, when a galvanometer is used under different conditions sensibility will be proportional to $T^2\sqrt{R}$. This formula, or others similar to it, has become familiar[1] as a means of ranking galvanometers by comparing their supposed performance under like conditions. Derived as it is from a consideration of the peculiarities of the Kelvin type it gives misleading results when applied to the moving coil galvanometer, for three reasons.

1. The sensibility of the moving coil instrument is not affected by the internal resistance, except as this modifies the total resistance. This is due to the fact that we can compensate for lack of wire in the coil by increasing the strength of the magnet — a thing usually impossible with the Kelvin type. (See equation (16), below.)

2. While a change in Q will of course alter the sensibility of a moving coil galvanometer, just as of any other, yet it will also destroy the condition of critical damping. The work required for the deflection varies as $\vartheta^2 Q$, and therefore, by (2), as $1/Q$ for given current, and the increased work necessitated by a reduction in Q

[1] W. E. Ayrton and T. Mather, Phil. Mag., 46, p. 350, 1898.

can therefore not be provided for by increasing the period only as $\sqrt{1/Q}$. The correct result is shown in equations (10), (12).

This objection does not apply to the Kelvin instrument, because that, being magnetically underdamped, is necessarily inefficient; a large part of the energy of its current is usually wasted, and we are at liberty to recover as much of it as we can. Increasing the period does this, as appears from (14).

3. The assumption that the sensibility of a moving coil galvanometer can be increased at all by diminishing Q is in many cases practically an impossible one. The limit of skill in the Kelvin instrument is reached in making the magnet system small; there is usually no difficulty in reducing the controlling force to correspond. The coil in the other type, however, is never as small as could easily be made, since the limit to progress in this direction is set by the relatively stiff suspending wire. The only way to increase the period of an instrument of maximum sensitiveness, therefore, is to make the coil heavier. Accordingly, the formula for comparing sensibility should be one in which K may be varied at will, but in which Q is fixed — that is, formula (11).

From this formula it appears that under the same external conditions the sensibility varies inversely as $\sqrt{Q}$ — that is, it depends ordinarily simply and solely on the size, elasticity, and length of the suspending wire. On the other hand, when T is diminished the sensibility diminishes only as the square root of T. The moving coil galvanometer is thus relatively more sensitive than the Kelvin at short periods.

Jaeger[1] calls attention to this difference between the two instruments. It may be worth noting, in regard to his and similar comparisons, that a Kelvin galvanometer is most serviceable when the whole resistance of the circuit is double its own, so that a reduction of such a galvanometer to a one-ohm resistance is equivalent to a reduction of the circuit to two ohms. Accordingly, since R in the moving coil formula applies to the whole circuit, it should be taken twice as great as the resistance of a Kelvin galvanometer in making comparison between the two — equal to 2 ohms in reckoning "normal sensibility." Hence Jaeger's comparison falls short of doing justice to the moving coil instrument.

[1] W. Jaeger, op. cit., p. 359.

This question is independent of the further fact, to which Jaeger calls attention, that since it is not necessary in the moving coil instrument to add wire in order to increase the value of G, its resistance should be made not equal to the rest of the circuit, but as low as possible. [1]

4. Wide vs. Narrow Coils.

(*a*) *As to Sensibility.* — The effect of coil width on sensibility has been the subject of an amount of discussion which is rather surprising, considering that, as shown by the formulæ already given, coil width does not necessarily have any effect on the sensibility. This is due to the fact that it is the value of the dynamical constant, $G(=Hlr)$, that is important, and this can, in general, be kept constant by modifying H in case l r changes. The contrary opinion seems to be based mainly on a demonstration by Mather,[2] which has been widely quoted. A glance at this demonstration shows that in it H is assumed constant, an assumption evidently unnecessary. Another assumption, which lurks concealed in the demonstration, is that the galvanometer is underdamped, since otherwise there is no object in increasing G. Moreover, as will be shown presently, the increase of G here obtained by narrowing the coil can also be produced by diminishing the size of wire. Mather's demonstration, then, amounts to this: In an underdamped galvanometer, where the field cannot be made stronger nor the wire in the coil smaller, it is well to increase G by making the coil narrow.

b. As to Resistance. — If the resistance per unit length of insulated wire is taken inversely proportional to the mass, the approximation is much closer than in the more familiar case where the ratio of resistance to cross section is considered. Then if m is the mass per unit length, the resistance per unit length is c/m, where c is a constant,[3] and x the resistance of the coil.

$$R' = \frac{lc}{m}. \tag{15}$$

[1] These two sources of gain are mutually exclusive, which is perhaps the reason why Jaeger omits mentioning one of them. But in any case, unless the galvanometer furnishes more than half the total resistance, the gain is at least as great as indicated above.

[2] W. T. Mather, Phil. Mag., 29, p. 434, 1890.

[3] c was found to be .000023 for a sample of 2-mil wire (diameter, .005 cm.), the units being ohms, grams, centimeters.

Moreover, we have [(4) and (5)] $K = lmr^2$, and $G = Hlr$. Eliminating m and l, we get:

$$G^2 = \frac{H^2 R' K}{c}. \qquad (16)$$

If we consider alterations in the coil which do not affect the principal constants of the instrument — period, sensibility, circuit resistance — we have: $H^2R' =$ constant. That is, field strength and coil resistance may be regarded as mutually replaceable elements; for a given value of G either one of them may be diminished at the expense of an increase in the other. Here, again, the result is independent of the shape of the coil.

The increase of resistance here contemplated can be effected in two ways; by making the coil narrower, thereby (4) increasing l, or by using more turns of finer wire. Hence, electrically, a narrowing of the coil is equivalent to a decrease in the size of wire.

The last conclusion suggests that a narrowing of the coil may be important when the finest possible wire is already in use and a further increase in the power of the coil is desired. This is, of course, the special but important case treated by Mather.

5. Detrimental Magnetic Impurities.

Ayrton has pointed out[1] that the magnetic impurities in the coil are acted on by the field with the production of a directive force which may greatly reduce the sensibility. To diminish this difficulty coils are usually made of materials as pure, magnetically, as possible. The detrimental effect can never be wholly eliminated, as has been shown in the case of the radiomicrometer. It is partly, if not wholly, due to residual magnetism, since a uniform field will not exert any disturbing action on a coil unless that coil is already magnetized in a direction different from that of the field. Accordingly, investigation of the detrimental effect seemed to require direct experiment. Measurements of the effect were made under varied conditions. The average error of the latest and best determinations was less than 1 per cent. The conclusion reached was that for commercial copper wire, and some other feebly magnetic

[1] W. E. Ayrton and T. Mather, Phil. Mag., 42, p. 442, 1896; Elect. Engineering, Oct. 5, 1894.

substances, in uniform fields, the detrimental magnetic effect is proportional simply to the square of the field strength and to the mass of the coil. Neither width nor shape had any perceptible influence, nor the direction of the wires, whether vertical or horizontal.

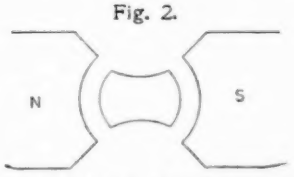
From these results it readily appears that if we consider changes which affect the coil alone, wide coils, being lighter, will give less trouble from magnetic impurity. In this case the wide coil will have finer wire. If the width of the coil is altered without changing the size of wire the magnetic effect also is unaffected, for in this case it is easy to show from (4) and (5) that the change in H offsets change in mass.

The coil is thus in unstable equilibrium, and as it rotates the couple tending to displace it further increases at about the same rate as the force of restitution due to the twisting of the wire.

6. The Radial Field.

The detrimental magnetic action in a uniform field is purely a directive force. Therefore in a radial field—that is, a field whose direction is at every point perpendicular to the direction of motion of the coil as it rotates—the detrimental effect should be largely eliminated. Such a field can be approximately realized by the arrangement shown in Fig. 2 (where the amount cut out on the sides of the central armature is probably a matter of minor importance). Its usefulness was tested in the following way: A coil, made sufficiently magnetic by a coating of soft wax and carrying a mirror, was suspended by a phosphor bronze wire, first in a radial field and then in a field of such uniformity that the strength varied less than one per cent. from the center to the side. The directive force of the field was found by observing the position of the coil with the magnet unexcited, and then noting the change when the current was put on. This was done for different initial positions of the coil. Repetitions of the same determination seldom varied more than one per cent. In general, with the radial field, a slight deflecting

Fig. 2.



Horizontal section of a radial field. The lines are everywhere perpendicular to the motion of the coil, so that this is unaffected by magnetic impurity.

force was found. This force can be compensated by twisting the suspending wire, and if constant is of practically no importance. It is the variation of the force as the coil turns which counts. That produces the effect of a directive force resisting the rotation. This variation in the case tested was less than one sixtieth the directive force of a uniform field of the same strength. The ratio would vary in different cases, depending on the accuracy with which the poles and armature were constructed, etc., but it is evident that the magnetic trouble may be very greatly diminished by the use of a radial field.

7. Description of a Galvanometer.

An application of the above formulæ was made in the construction of a galvanometer of relatively high sensitiveness, intended to be used in a circuit of from 1 to 100 ohms' resistance, especially with null methods. It seemed best to limit the length of the suspending wire to 10 cm., both for compactness and for low resistance. The finest wire obtainable was some listed by Hartmann and Braun as .0208 mm. in diameter. Silver was chosen on account of its low resistance. The moment of torsion of the upper and lower wires together was .17 c.g.s. units, and the resistance, 10 ohms. To secure a radial field of the proper proportions the least admissible coil width was thought to be 12 mm., and this was selected. The length of the coil was made 22 mm., in order to reduce sufficiently the effect of the dead wire, which now contributes about one fourth of the whole resistance, and one sixth the moment of inertia. On account of the intended use in null methods a period of 1 second was first planned, but it was found on calculation that this would involve a value of H so great as to make an electromagnet necessary. To avoid this complication the period was lengthened to 1.3 seconds, and for the same reason the number of turns of wire in the coil was made rather large (47) at the cost of a somewhat increased resistance. The total resistance is 36 ohms, and it could not well have been made less than half this, as long as the suspension was not changed. The coil weighs 100 mg. The field can be produced by either one or two permanent magnets, and its maximum value is about 900 lines. The value of Q, the moment of torsion, is about one fourth, and the sensibility, therefore, about twice that

which Jaeger[1] assigns as the probable limit at the present time. This result, however, is merely the natural consequence of using the fine wire.[2] It involves no special difficulties, for the suspension is amply strong, and has stood some unreasonably severe shocks without once breaking.

Several special features of the galvanometer may prove interesting. The radial field has already been mentioned. In an earlier galvanometer the presence of magnetic impurity nearly doubled the controlling force, and rendered the zero very inconstant. In the present instrument no shift of zero can be detected after ordinary deflections, nor any variation in the time of swing due to removing or replacing the magnets.

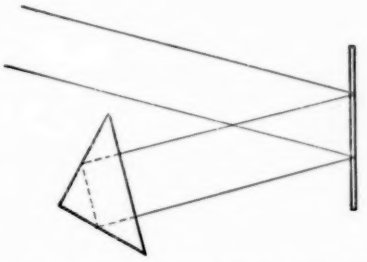
The coil is kept in shape by being bound with silk and varnished with collodion, a substance comparatively free from magnetic impurity. Coils 2 cm. square, and as light as 20 mg. can be made in this way.

The coil and suspension wires were permanently soldered together, and mounted on a glass rod, by means of which they are handled.

Ordinary solder seems to be a solvent for silver, and it is difficult to use it on these fine wires without melting them off. Fusible metal seems entirely free from this difficulty, and leaves the joints much stronger.

The pole pieces project from the case, and different magnets can be slipped upon them. An electromagnet could thus be used at any time if desired.

Fig. 3.



Showing a total reflecting prism used so that the effect of horizontal deflections of a mirror is doubled, and of vertical, eliminated.

The deflection is doubled according to a well-known method,[3] involving the use of a stationary mirror, so placed that the light is reflected twice from

[1] W. Jaeger, op. cit., p. 358.

[2] I have found Q approximately .18 units in two of the Leeds & Northrup Co.'s galvanometers. The suspending wires, of phosphor bronze, had about 20 ohms resistance.

[3] F. L. O. Wadsworth, Phil. Mag., 44, p. 83, 1897.

the moving mirror. For the stationary mirror is substituted a right-angled prism, with its hypothenuse toward the incident light and its elements horizontal (Fig. 3). Such an arrangement has two advantages: (1) the reflection is total; (2) such a prism behaves like an ordinary mirror toward horizontal deviations of the beam of light, but in the case of small vertical deviations the light is always returned parallel to itself and therefore the effect of all oscillations of the mirror in a vertical plane is neutralized. As magnet- and other suspended systems when disturbed by tremors of their supports frequently have their most energetic oscillations vertical, this method of eliminating such oscillations may, perhaps, be found generally useful. It is not important with a coil as steady as the one used in this instrument.

The scale is at two meters' distance. The mirror is 7 mm. wide; its moment of inertia is only one eighth that of the coil. This width of mirror, according to Einthoven,[1] would admit of a scale distance three times that actually used. As the instrument has ample sensibility for its present work, I have not sought to reach the limit in this direction.

The actual working sensibility of the present arrangement is 450 megohms at 62 ohms' resistance. This is .3 as sensitive as the sixteen-coil galvanometer at the Astrophysical Observatory (for equal period and resistance) and a little more sensitive than Dubois and Rubens' most sensitive "ironclad" galvanometer.[2]

8. Further Increase of Sensibility.

The above results are such as can be easily obtained with materials now available. It remains to inquire what further gain may attend special labor or skill.

1. A diminution in the size of the suspending wire would seem promising in the light of the large factor of safety evidently possessed by the wire of .02 mm. It seems unlikely that finer wire can be drawn directly, but if it could be drawn inside a copper coating, as platinum is in silver, I have been able to show that the copper could be removed by electrolysis at .2 volt without injuring the

[1] W. Einthoven, Drud. Ann., 12, p. 1062, 1903.

[2] H. Dubois and H. Rubens, Zeitschr. Instkunde, 20, p. 77, 1900.

silver. Whether, given the wire, the increase in sensitiveness would be nearer four- or fifteen-fold would depend on various circumstances. The gain would be considerably less if the suspension were shortened so as to keep the same resistance. Moreover, an increase in period would be almost necessary, as the small value of lr in a very light coil would require inconvenient values for H — unless indeed, the radial field were given up, and the coil made narrow, and this would introduce other difficulties.

Platinum wire is of course obtainable of sufficient fineness, but its rigidity and resistance are such that for low resistance it is no better than sizes of silver now available, while none of these extremely fine wires can carry a coil large enough to use in a high resistance circuit.

2. The suspending wire may be left as it is, and its torsion counterbalanced by another force, very much as one magnet of an astatic pair balances the other. The simplest way to do this is to make the coil one-sided, and hang it in an inclined position with the heavy side uppermost.[1] There are also other and better, because more stable, methods of accomplishing the same result.[2] Somewhat varied trials of two of these led to the conclusion that it is easy to increase the sensibility about three times, and apparently impossible to go much further. The main difficulty appears to lie in the imperfect elasticity of the wire, which renders all these methods practically useless, as every time the zero of the wire shifts the other element of the pair has to be adjusted to correspond.

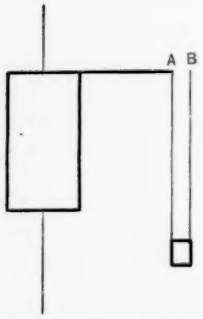
3. The relatively heavy and stable moving coil naturally lends itself to a certain amount of optical magnification. The limits of this, however, are soon reached, since the resolving power of the mirror (and therefore the permissible scale distance) increases as its breadth, while its moment of inertia increases more rapidly than the cube of the breadth.

4. The main disadvantage of the moving coil galvanometer is the necessity of passing a current into the moving portion, which compels the use of relatively stiff wire. There is no need of passing a current into the mirror, however, nor, if this is suspended separately,

[1] H. Abraham, L'Éclairage Électrique, 7, p. 276, 1896.
[2] W. Stroud and J. H. Oates, Phil. Mag., 6, p. 711, 1903.

is there any lack of energy for turning it. A method due to Kelvin has been used by Prof. C. E. Mendenhall to produce a large deflection of a mirror from a small movement of the coil. The device is shown in Fig. 4, where the mirror is suspended by fibers from the points A and B, one of them fixed, the other at the end of an arm carried by the coil. This arrangement evidently requires great stability, for it responds to movements of translation, as well as rotation. This difficulty is avoided by the arrangement shown in Fig. 5. The mirror is suspended by the single fiber, A, so as to be drawn slightly forward against the two fine hooks, H, K, which are connected with two corners of the coil. I have tested this method (without the hooks) on a blank coil; it works well, and is extremely stable. It did not occur to me till after the galvanometer was finished.

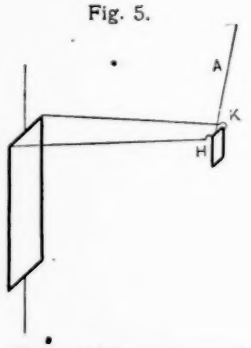
Fig. 4.



Device for multiplying the motion of a coil by a separate mirror.

Since the energy of a rotating body is proportional to the square of the angular velocity, the work required to turn a separate mirror will be multiplied by the square of the ratio in which the sensibility is increased. The moment of inertia of the mirror must be correspondingly diminished if the work expended in turning it is to be kept constant. For instance, a mirror whose motion is multiplied ten times should have one hundredth the moment of inertia of one attached directly to the coil. That would give, in the case of the galvanometer described above, a mirror width of 2 mm. The corresponding sensibility, using the reflecting prism, would be five times what it is at present, and greater than could be obtained by any of the other methods here described.

Fig. 5.



More stable arrangement for the same purpose. The mirror is held at three points and is not rotated by translatory motion of the coil.

Increasing the size of the coil has precisely the same effect upon the allowable multiplication as decreasing the mirror — that is, the

multiplication varies as the square root of the moment of inertia of the coil. It is thus (12) inversely proportional to the sensibility of the coil. The sensibility attained by the mirror, then, is constant — that is, it depends upon the mirror and not at all upon the coil. This result also follows directly from the assumption used in getting it — viz., that a certain proportion of the energy should be expended on the mirror.

The size of the coil would really be immaterial if there were no question of stability involved. But in practice a greater ratio of multiplication would involve a greater magnification of the tremors of the coil. Since, however, rotations of the coil are the only motions which will trouble the mirror appreciably, and since a large coil can be balanced more truly than a small one, there is evidently a point beyond which increased multiplication is preferable to a decrease in the size of the coil. It appears not improbable that the fitting of a multiplying device to such galvanometers as are now on the market may usually be the most advantageous method of securing high sensibility.

9. The String Galvanometer.

An ingenious modification of the moving coil principle is found in Einthoven's recently described "String Galvanometer," [1] in which the advantages of a mirror are given up, and the resulting loss more than compensated by the exceeding lightness of the moving system thus made possible. The galvanometer consists of a silvered quartz fiber, 2.3 μ in diameter, stretched like a piano string in a field of 2,300 lines, its movement magnified 660 times by a microscope.

This instrument is easily brought within the scope of the above equations. (Not those for critical damping.) If we assume that the fiber, fixed at the ends and swinging in the middle, is equivalent to a straight fiber of half the length moving uniformly it then corresponds to a coil of this length and equal weight, of 3 mm. radius, since a coil of that radius would have its linear motion magnified 660 times by a mirror with a scale at a meter's distance. The silver, estimated from its resistance, 10,000 ohms, would appear to furnish about one twentieth the volume of the whole fiber. The

[1] W. Einthoven, Drude Ann., 12, p. 1059, 1903.

average density of the fiber may then be taken as not far from 3, which makes its weight, in round numbers, 1×10^{-6}. The greatest sensibility obtained by Einthoven is 100,000 megohms. This gives, in accordance with equation (2), $Q = 8 \times 10^{-5}$, taking $G_1 = H/r$, = 41,000. This gives for $T/2$, .1 second. The observed value was 20 seconds, two hundred times as great, which shows a truly enormous amount of air damping. This damping gives the readings great steadiness, an important advantage of this form of galvanometer. At the same time the advantage of the very light moving system is so far neutralized that the sensibility is, considering the resistance, lower than that of the best Kelvin galvanometers. This is very far, however, from saying that the instrument is less useful. At shorter periods, as might be supposed, the air damping is less important, and the sensibility surpasses all other galvanometers.

On the basis of the performance at one two hundredth of a second Einthoven computes a normal sensibility — that is, a sensibility at one ohm and five seconds — of from 900,000 to 2,380,000 megohms — more than he was able to get at 20 seconds with 10,000 ohms. This discrepancy between theory and practice illustrates the idea suggested on page 318, above, that the normal sensibility of a galvanometer has little value when computed in accordance with the properties of a radically different instrument. Einthoven takes pains to make his computation, first, on the assumption that the normal sensibility varies as the square root of the resistance, and then that it varies as the three-fifths power. Now the question which of these two ratios should be used has to do, among other things, with the thickness of the insulation of wires. It originated in a discussion of the Kelvin galvanometer, and was in its origin a thoroughly practical question, resulting from the attempt to make the formula conform as far as possible to the physical conditions obtaining in the construction of that instrument. But whether the ratio of reduction should be the one half or the three-fifths power is surely a minor consideration in the case of an instrument where the sensibility actually varies more nearly as the first power. (The justification of this last statement is found in the fact that in this galvanometer a decrease of resistance is in general accompanied by an increase of the moving mass.)

10. Stability — The Julius Suspension.

Any device, optical or other, for magnifying the deflection of a coil manifestly depends for its success on a high degree of stability. Accordingly, the work on the galvanometer above described has been accompanied by an attempt to secure a specially stable support.

The most popular method of securing stability at present is undoubtedly the Julius suspension,[1] in which the galvanometer support, hanging from long wires or cords, is only slightly affected by lateral tremors of the building to which it is attached. This very simple and efficient arrangement has two disadvantages: (1) The apparatus is liable to be set swinging by air currents; (2) there are always oscillations about a horizontal axis. A good deal of attention has been paid to these latter. Since they are oscillations about the center of gravity their effect is largely neutralized by attaching at that point the fiber which bears the magnet system. This adjustment is one of the special points of the Julius suspension. It is evidently desirable to diminish these oscillations also. To secure this result, Julius directs that the suspended frame be made symmetrical, and the suspension wires as nearly alike as possible. The impulses coming down the different wires will thus be very nearly alike, and so will have the least tendency to produce rotations. The absence of this condition Julius assigns as the probable cause for the failure of attempts to secure stability by suspending from springs, etc.

There are, nevertheless, reasons for thinking that springs may be an advantage. The equal wires are never able to exclude altogether the impulses that produce rotation, and those which do arrive act upon a system capable of very energetic rotation with vibrations of a period comparable with those of the disturbances. If the system were made incapable of these vibrations it would be much steadier, even if the arriving impulses were somewhat increased. Hamy,[2] who has also treated the question mathematically, has obtained good results with springs, but finds that some precautions are necessary to get the best results. Damping devices are

[1] W. H. Julius, Wied. Ann., 56, p. 151, 1895; Zeits. Instrkunde, 16, p. 267, 1896.
[2] M. Hamy, Compt. Rend., 136, p. 990, 1903.

especially important. The neglect of some of these precautions may account for the failures alluded to by Julius. Suspensions supported by springs are reported as "very efficient" at Johns Hopkins University. I have found that three different systems were improved by springs, especially if these were long. Some years ago a very crude suspension in use at Cornell University, in which the magnet system was not well protected from the oscillations about the center of gravity, was improved by the substitution of cords for wires, and still further when the cords were saturated with linseed oil. Here there seems to be little doubt that the improvements were due to the elasticity of the cord, and the damping effect of the viscous material. In all such cases discrepancies between the results obtained by different observers are apt to arise from the fact that there are, so to speak, various kinds of stability, and different apparatus and conditions require different treatment.

The Julius of course answers all ordinary requirements[1] — the above discussion applies rather to the problem of securing specially high stability. When this is desired, however, there is an evident profit in making some effort to secure the steadiest possible foundation to begin with. This will generally be a basement floor; accordingly, the writer tried to make use of devices which did not need to be supported from above.

11. Stability Obtained by Flotation on Mercury or Oil.

This method has been studied by Einthoven[2], who used mercury. Although his results as announced bear no apparent resemblance to those of Julius, a little examination shows that the properties of the two tremor-shielding devices are essentially similar. Einthoven found that for good results the float should be flat and the apparatus low down upon it. The flat float, offering a small lateral surface to the fluid, acts like the long wires of the Julius in conferring freedom from purely horizontal disturbances. That is, upon the center of

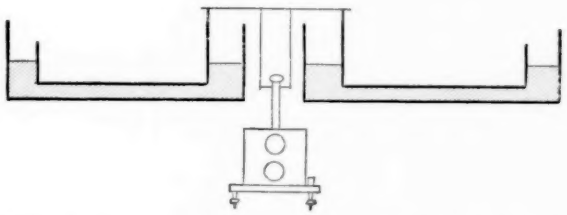
[1] Dr. A. L. Day has called the writer's attention to the fact that, in addition to the adjustments described by Julius, the regulation of the dash pots is important — so much so, that changes in the temperature of the room, by altering the viscosity of the damping fluid, may materially affect the stability.

[2] W. Einthoven, Wied. Ann., 56, p. 161, 1895.

gravity; since the float must follow the vertical motions of the support at every point it will rotate like the Julius suspension (though, in general, to a less extent), and points above or below the center of gravity will have horizontal motion. Putting the apparatus to be shielded low is simply putting it near the point of least motion.

It would manifestly be still better if the apparatus could be put exactly at the center of gravity, as in the Julius suspension. This result can be secured by making both tank and float in the form of hollow squares, as shown in vertical section in Fig. 6. As the

Fig. 6.



Tremor-shielding device. Vertical section through a tank and float, made in the form of hollow squares so that apparatus can be supported at the center of gravity, the point of least disturbance.

apparatus is now suspended from the float, it may be of any shape we please.

Another alteration would be the substitution of heavy mineral oil[1] for mercury. By this several advantages are gained. The oil is cheaper, and non-poisonous; on account of its viscosity it tends to damp out the waves produced by tremors of the tank; it does not require so careful leveling, on account of the thicker layer admissible underneath the float, resulting both from the greater cheapness of the oil, and its greater damping power.

Comparing this arrangement as to its efficiency with the Julius, we have the tank offering to the oil a lateral surface not much greater than would be exposed in a set of dash pots, and this is, with the exception of the air, practically the only medium for communicating tremors to the float. It might be expected to behave, therefore, like a Julius system in which the horizontal disturbances from the dash pots were slightly increased, and those from the

[1] D. B. Brace, Astroph. Jour., 5, p. 214, 1897.

wires entirely removed, while the oscillations of the support about a horizontal axis were not augmented, as they are apt to be with the Julius. The tank takes up more floor space, and instruments on it are less accessible. It costs less, and can be more easily shifted from one part of the room to another. It is especially adapted to that large class of cases where a pier is present, or where the floor itself is steadier than the ceiling.

Abbott, at the U. S. Astrophysical Observatory,[1] used a combination of wire and fluid support, putting the float at the top. There are two advantages in putting it below. (1) We can use either larger masses or a smaller amount of fluid, since the float has only the galvanometer to sustain. (2) The instrument will remain level in spite of changes in the suspension wires. This latter advantage, indeed, might prove a powerful reason for the use of the flotation method, aside from its additional tremor-shielding power, in cases where the use of springs caused troublesome changes in the level of the Julius alone.

12. The Jointless Table.

If a rod or bar of wood or metal is stood upright on one end, then for any small horizontal vibrations of the supporting surface the center of percussion of the bar — two thirds of the way up, if it is uniform — will have very little horizontal motion. If three equal upright bars have cleats fastened to them at the center of percussion a board or frame laid upon the cleats will form a table which is relatively independent of tremors. Such an arrangement, like the tank and float, is evidently most useful where the floor is steadier than the walls or ceiling. It may possibly be the same arrangement which Abbot calls a "rickety table,"[2] but I am not aware that it has been particularly described.

One great advantage of the arrangement is that, like the fluid tank, it is self-damping. It needs to be guarded in some way against accidental overturn, but when it is so protected apparatus can be adjusted on it without difficulty. When made of wood it is the quickest to make and set up, and the most portable, of all tremor-

[1] C. H. Abbott, Astroph. Obs. Annals, 1, 62, 1900; Appendix to Report of Sec'y of Smithsonian Institute, p. 100, 1900, p. 122, 1901.

[2] C. H. Abbott, Smithsonian Report, p. 102, 1900.

shielding devices. For continued use it should be of metal, as the warping of wood gives endless trouble. In the preliminary tests of the jointless table, it was found that an ordinary table magnified the tremors of the floor even more than the other diminished them. Hence, if a table or stand must be used on a pier the substitution of a jointless table for an ordinary one gives a specially great gain.

13. Experimental Results.

The three forms of support just described were tested, singly and in various combinations. An exact comparison of tremor-shielding devices would require that they be supported at identically the same points, and then the results would apply to the particular apparatus used to test the amount of shielding. No such exact comparison was attempted, and the conclusions are somewhat in the nature of estimates. The steadiness was judged in all cases by the use of mercury, and usually by various galvanometer systems also.

The estimate made above of the relative affectiveness of the Julius and the tank seemed confirmed. The center of gravity of the tank was much steadier than points not 20 cm. above it, though the difference was less than with the Julius, which seems to magnify the oscillations which its support performs about a horizontal axis.

The combination of a tall jointless table supporting a Julius does not seem worth while under ordinary circumstances, considering the liability to disturbance from air currents and the complication generally. On the other hand, a low jointless table seems less effective than either of the two other supports. Nevertheless it would be superior to the Julius if the floor were more stable than the ceiling. It is also an advantageous support for a tank.

The stability finally attained was limited by the effect of air currents. The disturbance from them is so great that it has become necessary to use nothing but the float (which is somewhat protected by the walls of the tank around it), and to hold this in place by relatively stiff springs. That is to say, since the whole object of a tremor-shielding arrangement may be said to be rendering a piece of apparatus independent of the building in which it is, the air currents have in this case compelled a deliberate sacrifice of the steadiness otherwise attainable. Even as it is, while a violent

stamping on the floor near the apparatus produces no perceptible effect, the waving of an arm at the same distance gives a millimeter deflection. This result is partly chargeable to the galvanometer, which is but slightly affected by tremors, but has, on account of its damping being mostly magnetic, small protection against the production of a pendulum-like swaying of the coil, caused by slow motions of its support. In any case, it is clear that one condition of high stability in the presence of tremors is a very thorough shielding against air currents. The necessity for such shielding is often mentioned, but the extent of it may prove surprisingly great. A Julius support carried on a jointless table was markedly disturbed if anyone walked by at three meters' distance, outside a closed door. There was no question that the effect was due to the movement of the air, and not to ground tremors.

14. SUMMARY.

1. In all galvanometers a counter E.M.F. is produced by the motion of the coil or magnet, which seems to cause a damping effect. When there is no appreciable waste of energy and when the counter E.M.F. is sufficient to make the motion just barely aperiodic, the galvanometer has the maximum possible effectiveness, as far as utilizing the energy of the current is concerned, and this effectiveness is the same for all such galvanometers.

2. The power of the magnet in moving coil galvanometers renders easy the securing of the above condition, in ordinary cases.

The sensibility for given resistance and period then depends solely on the deflection produced by a given quantity of energy. Moreover, for such moving coil galvanometers as are generally used the limit of delicacy of construction is set by the size of the suspending wires. It follows that the normal sensibility depends solely on the directing force of the coil, and is proportional to the square root of that.

3. When resistance or period is changed the sensibility is proportional to $\surd RT$, where R applies to the whole circuit. To employ instead of this formula the one derived from the Kelvin galvanometer gives misleading results.

4. A further result of the magnet power of the moving coil gal-

vanometer is the ability to replace part of the wire of the coil by other masses without loss of sensibility. This type is thus by far the best when pointers, etc., are to be moved, as in lecture table apparatus.

5. For the same reason the width of the coil has, in general, no effect on the sensibility.

6. A change in the width of the coil has, electrically, the same effect as a change in the size of wire in the coil.

7. Coil resistance and magnet strength may to a large extent be substituted one for the other. The dynamic constant is proportional to the product of the field strength by the square roots of the coil resistance and of the moment of inertia.

8. Trouble arising from magnetic impurities in the coil is reduced to one sixtieth by using a radial instead of a uniform field.

9. It is easy to make a simple and "robust" moving coil galvanometer that shall have an *effective* sensibility of 560 megohms for 100 ohms resistance and a period of 1.3 seconds. (This slightly excels the most sensitive Dubois-Rubens ironclad galvanometer, for the same conditions.) The internal resistance can not then be brought much under 20 ohms.

10. A separate mirror promises a very considerable increase in this sensibility.

11. Attempts to increase sensibility by decreasing the control below the limit naturally set by the wire have not been successful.

12. In securing high stability a device which can be supported by a basement floor is preferable to any arrangement, in itself equally good, which hangs from a wall or ceiling. A tank and float may thus be of greater advantage than a Julius suspension.

13. It is very much better, when a float is used, to suspend the apparatus from its center of gravity. This is readily accomplished by making the float and tank annular.

14. A jointless table permits taking advantage of a stable floor, and excels in simplicity and portability.

CORNELL UNIVERSITY.

TEST OF THE LIQUID AIR PLANT AT WESLEYAN UNIVERSITY.[1]

BY W. P. BRADLEY AND A. W. ROWE.

THE plant was installed in the summer of 1902. It consists of a 15 H. P. Mietz and Weiss kerosene motor, a four-stage compressor built by the Norwalk Iron Works Company, of South Norwalk, Conn., and a liquefier constructed in our laboratory.

The cylinders of the compressor are single acting, and are 7¼, 3⅞, 2, and 1 inch in diameter respectively. The air is taken from under the eaves of the building into a forty-gallon tank or reservoir, and thence directly into the low pressure cylinder. The compressor is provided with three intercoolers in the usual way.

From the compressor, the air passes successively through an aftercooler; a separator; two cylinders, containing potassium hydroxide in sticks and calcium chloride respectively, in order to remove the carbon dioxide and water vapor; and finally through a precooler charged with broken ice or snow — reaching the liquefier at a temperature of about 2°. The unliquefied air which escapes from the liquefier is carried back through the reservoir already mentioned into the compressor again.

There is an obvious advantage in using over again the air which has once passed through the purifiers. As it is practically impossible however so to adjust the valve of the liquefier as to accommodate exactly the amount of air which is being delivered from the compressor, it frequently happens that this purified air returns for a short time somewhat too rapidly from the liquefier to be taken up at once by the compressor. The reservoir is intended to store this excess until the next adjustment of the valve enables the compressor to overtake it.

The liquefier is constructed of seamless drawn copper pipe, of 4.8 mm. outside diameter and .8 mm. thickness of wall. Two

[1] Contribution from the Cryogenic Laboratory of Wesleyan University.

lengths of this pipe, of somewhat more than 50 meters each, are wound in parallel so as to form a series of discs, fifty-nine in number, lying one above another and forming a cylindrical column 11.75 cm. in diameter and 41 cm. in height. A space of 1.6 mm. separates the discs and an equal space separates the turns of pipe in each disc.

Two types of liquefier are in somewhat general use. In one of these, that of Linde, the current of compressed air is divided at the expansion valves, the greater portion returning to the compressor at an intermediate pressure, while the rest, minus the liquid formed, leaves the liquefier at atmospheric pressure. In the Hampson liquefier, on the other hand, all the air which passes the valve drops at once to atmospheric pressure. The liquefier here described belongs to the latter, or single-circuit type.

The measurements of the various factors involved in the tests were made as follows:

The horse-power developed in the engine was taken in the first instance from indicator cards in the usual way. At the close of the tests, the accuracy of this indicated horse-power was checked once for all by a brake test, during which cards were taken.[1]

The kerosene used in the motor had a specific gravity of 0.775, and its combustion value was 11.072 calories.[2] Accordingly, one quart of the oil represents 12.66 H. P.

To measure the amount of air delivered by the compressor, an arrangement was made whereby the air current could be diverted at will from the liquefier into a large metallic cylinder or eudiometer. The method is illustrated in Fig. 1. The valve of the liquefier having been closed, the throttling valve A is opened and so adjusted

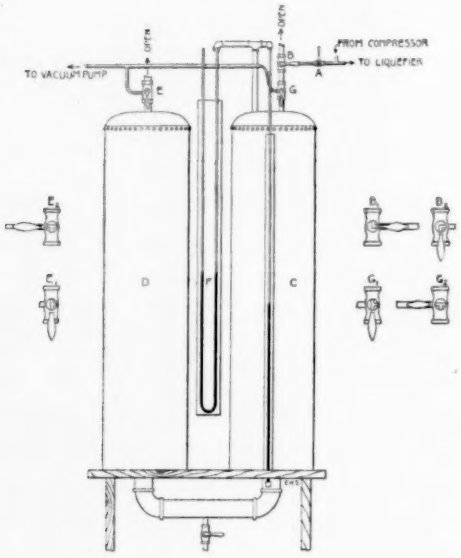
[1] Two indicators were used during the tests. They were obtained respectively from the Star Brass Manufacturing Co. and from the Crosby Steam Gauge and Valve Co., both of Boston, Mass. The former was of the usual type, including the inside spring, while the latter was of a new design with the spring mounted above the arm. Both were accurate and gave concordant results. The new indicator is somewhat more convenient for use upon an explosive engine, as it is more easily taken apart for cleaning.

[2] We are indebted to Mr. Ernest M. Swett for this determination of the thermal equivalent of the oil, which was carried out with the aid of the Atwater modification of the Berthelot bomb calorimeter. Three tests were made, giving the values 11.057, 11.092 and 11.066 respectively.

In the calculation of the horse-power equivalent, the U. S. quart is considered equal to .9464 liters.

that the high pressure gauge of the compressor remains stationary at the pressure required. During the establishment of equilibrium in this way, the valve B is maintained in the horizontal position B_1, and the blast escapes through it into the open air. When a measurement is to be taken, B is suddenly turned downward into the position B_2 for a measured interval of time, during which the air

Fig. 1.



flows through the valve G (position G_1) into the eudiometer C, forcing the water over into the reservoir D.

If a suitable time interval has been chosen, the water will have been nearly all driven into the reservoir and the air in C will therefore stand under a certain slight pressure, which will be indicated by the gauge F. To bring the air to atmospheric pressure before reading, the reservoir is connected with a small suction pump by means of the valve E (position E_2), until the liquid in the arms of the gauge F is leveled, the top of E being covered meanwhile by the hand.

To prepare for the reception of a new charge of air from the compressor, the suction pump is switched from D to C (with the valves in the positions B_1, G_2, E_1).

A carefully calibrated tell-tale tube on the outside of C shows the

volume of air delivered. Under each set of conditions employed, several concordant readings of volume were taken and averaged.

The approximate horse-power equivalent of the air delivered, which is designated below as compressor horse-power, is calculated on the basis of isothermal compression by the formula:

$$HP = \frac{P_1 V_1 \log_e P_2/P_1}{33{,}000}$$

where

P_2 = absolute pressure of compressed air in pounds per square foot,

P_1 = atmospheric pressure in pounds per square foot (14.7×144),

V_1 = volume in cubic feet per minute at atmospheric pressure, P_1.

The amount of liquid air produced was measured by drawing it into a large vacuum test-tube of about 600 c.c. capacity. It was drawn at regular intervals of six minutes, being allowed to accumulate in the reservoir of the liquefier between drawings.

However the liquid is drawn, a certain loss by evaporation is unavoidable. This loss is greater if the liquid flows continuously as fast as formed, rather than in full stream at not too short intervals. Of course the liquid evaporates also on standing, whether in the liquefier or in the vacuum vessel. Probably evaporation is more rapid in the liquefier. But it is worth nothing that while outside evaporation involves a total loss of the energy represented in the portion of liquid evaporated, inside evaporation does not. Even if it be granted that the heat required to convert liquid into vapor of its own temperature comes in part from without, that which serves to raise that vapor to room temperature is taken from the coils of the interchanger, and so from the incoming compressed air. In so far, at least, liquid which evaporates inside the liquefier tends to replace itself.

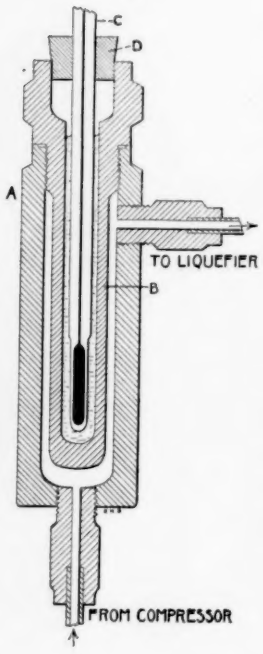
The yield of liquid, as recorded for any given set of conditions, represents the average of a considerable number of drawings. Of course no drawings were recorded until the liquefier had reached thermal equilibrium.

In calculating the horse-power equivalent of the liquid, its total

energy per gram was assumed to be 97.5 calories,[1] and its specific gravity, 0.932.[2]

During all the tests given in this paper, careful note was made of the initial and final temperatures of the air, as it was delivered to, and escaped from the liquefier. The final temperature is easily obtained by the use of an ordinary thermometer whose bulb lies in the direct path of the air as it emerges from the top of the interchanger. The initial temperature on the other hand gave us some trouble until the adoption of a simple device which is illustrated in Fig. 2, and which scarcely needs any explanation otherwise. The compressed air, just before entering the liquefier, passes through the block *A* in such a way as to completely envelop the pocket *B*, which contains the bulb of the thermometer *C*. A little water introduced into the pocket serves to convey the temperature of the air to the thermometer more quickly than would otherwise be the case. If the initial temperature were below 0°, kerosene or some other suitable liquid could be substituted. The thermometer is held in place by a rubber stopper *D*.

Fig. 2.



We pass now to the results obtained.

In Table I. are presented complete records of four runs, made at the ordinary working pressure of 2,600–2,700 pounds. One other run is included, in which no determination was made of the amount of air delivered by the compressor, and still another, in which no cards were taken from the motor. These two runs were made before the arrangements for the tests were complete. This table gives a fair representation of perhaps over one hundred runs made with the plant for the purpose of obtaining liquid air simply, and of which no record was kept except the amount of oil consumed, and the amount of liquid air produced, per hour.

[1] Allen and Ambler, PHYSICAL REVIEW, **15**, 1902, 183.
[2] Knipp, PHYSICAL REVIEW, **14**, 1902, 75.

TABLE I.

Normal Runs.

Pressure.	Air.		H. P.				Cu. Cm. Liq. per Eng. H. P.	Per cent. Air Liq.
	Cu. Ft. per Min.	Liters Liq. per Hour.	Oil.	Eng.	Comp.	Liq.		
2,665	14.9	2.84			5.0	.40		8.6
2,670		2.35	98.7	5.2		.33	452	
2,650	15.5	2.80	86.1	7.1	5.2	.40	394	8.0
2,650	14.4	2.60	83.6	6.3	4.8	.37	413	8.1
2,660	15.0	2.88	81.0	5.7	5.0	.41	505	8.7
2,670	13.6	2.66	83.6	6.2	4.5	.38	429	8.8
Av. 2,661	14.7	2.69	86.6	6.1	4.9	.38	439	8.4

A study of the data reveals the following facts:

The Motor. — The efficiency of the motor averages 7.1 per cent. This is a low efficiency for a motor of this type, but it is sufficiently accounted for by the fact that the load is less than half of that for which the motor was designed. It is well known that explosive engines, though they exhibit great efficiency when run at full load, as compared with those driven by steam, fall off in this regard with especial rapidity when they are either overloaded or underloaded.

The Compressor. — The horse-power equivalent of the compressor output averages 77.7 per cent. of the indicated horse-power of the motor.

It should be said in this connection that the compressor was run at a speed of 133 revolutions per minute, and not at its normal speed of 180. Something more will be said on this point below.

The efficiency of the compressor varies considerably in the different runs. Thus in the fourth, only 14.4 cubic feet were delivered, with an expenditure of 6.3 H. P. in the motor, while in the fifth, 15 cubic feet required but 5.7 H. P. Not all of the apparent variation is to be charged to the compressor, for the motor friction, and slip of the belt cannot be supposed to remain constant from day to day. A large share is due however to the compressor, as was shown by a study of the pressures registered by its three lower gauges. The details of the matter would be out of place at this time.[1]

[1] Piston leakage appears to be the chief cause of variation in the efficiency of the compressor from one day to another, and this in turn seems to be due to variations in the

The Liquefier. — The air liquefied averages 8.4 per cent. of the air delivered from the compressor.

The average yield of liquid air per horse-power per hour is 439 c.c.[1]

In the fifth test the yield is 505 c.c. This is 15 per cent. higher than the average, and is considerably the highest yield per horse-power per hour which we have ever obtained with this liquefier. In spite of that fact, there is considerable reason to think it more nearly representative of the normal functioning of the plant than the average yield itself. The record was made after the plant had been running over three hours in the production of liquid air. All of the peculiar fluctuations of pressure on the three lower gauges, which have already been alluded to as characteristic of the early stages of a run, had disappeared, indicating the establishment of thorough and uniform lubrication. The motor, which always runs somewhat stiffly when first started, had smoothed down and was consuming an unusually small quantity of oil. And the purifiers were working very perfectly, with the result that the liquid air, as it flowed from the reservoir, was entirely free from turbidity and sparkled like fresh spring water.

All of the other tests were made as soon as thermal equilibrium was fairly established in the liquefier, and therefore under comparatively less favorable circumstances. Indeed, in the third test especially the motor piston was known to be working with much friction, owing probably to the deposition of carbon in the cylinder, while in the fourth, the compressor gauges revealed the fact that the piston leakage was greater than usual during the entire run, so that we were not at all surprised on measuring the air delivered at the close

lubrication of the cylinders, particularly the third and fourth. These receive no direct, independent lubrication. The third cylinder depends upon the oil which is brought along mechanically from the second, while the fourth draws similarly from the third.

[1] Some previous records are:

Standard Hampson Liquefier: 340 c.c. (Hampson, Scientific American Supplement, July 1, 1899). 200–300 c.c. (Travers, Study of Gases, pp. 192–4). 166 c.c. (Olszewski, Zeitschrift für comprimirte und flüssige Gase, 7, 1903, 9–12). 124 c.c. (Allen and Ambler, Physical Review, 15, 1902, 184).

Olszewski's modification of the Hampson liquefier (loc. cit.). 166 c.c. without precooling. 330 c.c., precooling with solid carbon dioxide and ether.

Linde liquefier: 333 c.c., precooling with calcium chloride and ice (Linde, Zeitschrift für comprimirte und flüssige Gase, 1, 1897, 119). 250 c.c. at — 15° (ice and salt precooling) (Müller, Zeitschrift für comprimirte und flüssige Gase, 3, 1899, 91).

of the run to find it less than usual. Both of these comparatively unfavorable tests are presented nevertheless with the rest, because they represent conditions which are likely to be encountered at any time in connection with the short runs which are often made with the plant.

The average efficiency of the entire plant is 6.2, that is, the horse-power equivalent of the liquid air produced averages 6.2 per cent. of the indicated horse-power of the motor. In this respect also, as might have been expected, the fifth run is well in the lead, with an efficiency of 7.2 per cent.

The Effect of Initial Temperature.

Among the recorded yields at other laboratories, which have already been given, it will be noticed that the last three represent precooling through the aid of freezing mixtures. Müller used ice and salt; Linde recommends calcium chloride and ice; while Olszewski uses a still more powerful cryogen, solid carbon dioxide and ether. In all three cases, the yield is larger than the ordinary, as was to have been expected.

It seemed of interest to test the effect of initial temperature on the output of our liquefier. The results embodied in the following table were secured, so far as temperatures above 20° are concerned, by substituting hot water for the ice of the precooler. The desired temperature was maintained by leading into the water a suitable current of steam. The data were all obtained in a single continuous run, so as to secure as nearly uniform conditions otherwise as possible, and after each change of temperature the record began only when thermal equilibrium had been secured, and the drawings of liquid became concordant.

The pressure was 2,650 lbs. by the gauge. The horse-power amounted to 7.1, and though rather high, remained quite constant.

Diagram I. exhibits the manner in which the yield of liquid air per horse-power per hour decreases with rising temperature.

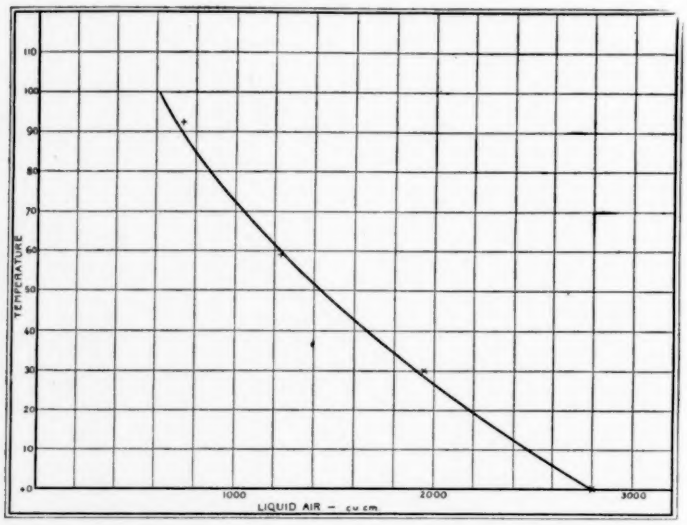
Whether at a sufficiently high initial temperature the liquefier would cease to produce liquid air cannot be decided by these data. Much less is it possible, if so, to form any conclusion as to the location of such a thermal limit. In any event, the great economy of

TABLE II.

Effect of Initial Temperature.

Initial Temp.	Air.		Eff. of Plant.	Per cent. Air Liq.	C.c. Liq. per Eng. H. P.
	Cu. Ft. per Min.	Liters Liq. per Hour.			
+2	15.5	2.80	5.6	8.0	396
30	15.3	1.94	3.9	5.7	274
59	15.0	1.23	2.5	3.7	174
92	15.0	.74	1.5	2.2	105

Fig. 3.



power which is effected by precooling is made very clear by the diagram.

We have as yet made use of no cooling agent other than broken ice or snow. It is our intention soon to extend our tests to low initial temperatures also. Even on the conservative assumption that the efficiency curve is rectilinear at temperatures below 0°, efficient precooling by means of solid carbon dioxide and ether, as employed by Olszewski should increase the average yield of our liquefier to five liters or more per hour, which would be equivalent to 700 c.c. per horse-power.

Effect of Initial Pressure.

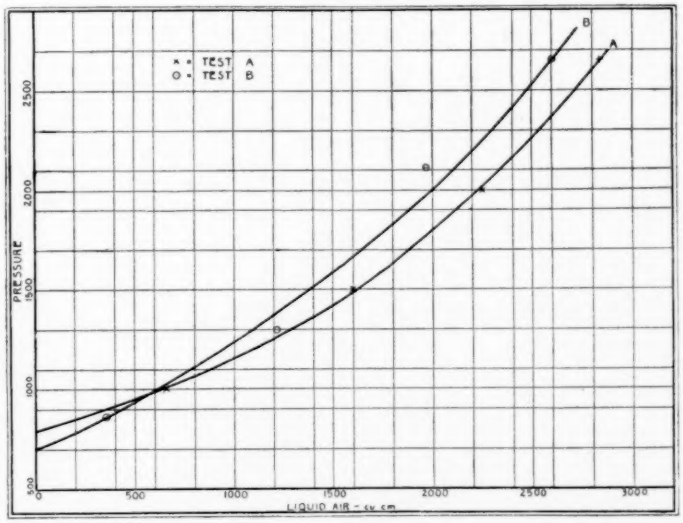
Another question upon which some data have been secured is the relation between the initial pressure of the air supplied to the liquefier and the yield of liquid air. Two representative tests are given, which were made about a year apart. In the first, no indicator cards were taken. In the second, the horse-power of the motor is given for the higher pressures only. When the compressor is being driven at pressures in the neighborhood of 1,000 lbs. or lower, the explosions in the cylinder of the motor become irregular, and it is

Table III.

Effect of Initial Pressure.

Test.	Pres.	Air.		H. P.			Eff. of Plant.	Per cent. Air Liq.	$t_1° - t_2°$.
		Cu. Ft. per Min.	Liters Liq. per Hour.	Eng.	Comp.	Liq.			
A	2,665	14.9	2.84		5.0	.42		8.6	0.0
	2,000	15.8	2.25		5.0	.33		6.4	0.25
	1,500	16.2	1.60		4.8	.23		4.5	0.50
	1,000	17.1	.66		4.6	.10		1.7	3.25
B	2,650	14.4	2.60	6.3	4.8	.38	6.0	8.1	0.0
	1,980	14.6	2.11	5.5	4.6	.31	5.6	6.5	1.0
	1,300	15.8	1.21		4.6	.18		3.5	2.5
	850	16.4	.37		4.3	.05		1.0	5.5

Fig. 4.



difficult to obtain cards which are fairly representative. Each of the tests, *A* and *B*, forms a continuous run, in which, after each change of pressure, no records were made until equilibrium was established throughout the plant.

Diagram II. exhibits the effect of pressure on the yield.

At low pressures the yield falls off very rapidly indeed. In fact, it is evident that not far from 700 lbs. no liquid would accumulate at all, though the temperature of liquefaction would be maintained. The cold produced at the valve would be exactly offset by the various unavoidable losses.

Two of the factors may be mentioned which contribute to the impaired efficiency of the liquefier at low pressures. In the first place, if the generally accepted view be correct,[1] that the fall of temperature at the moment of expansion is due to the Joule-Thompson effect, the degree of refrigeration produced at the valve should decrease with the pressure. In the second place, the larger volume occupied by a given mass of air at low pressure necessitates a much more rapid passage through the liquefier, inside the pipes. This rapidity of flow becomes increasingly an obstacle to perfect thermal interchange between the inflowing and outflowing currents. This is eloquently attested by the figures in the last column of the table, which show the difference between the temperature of the air as it enters the liquefier ($t_1°$), and that at which it escapes ($t_2°$). At the ordinary working pressure of 2,600–2,700 lbs. this difference is so slight as to be negligible, while at the lowest pressures it increases very rapidly.

At 850 pounds, for instance, the escaping air was 5.5° colder than when it entered the liquefier. Of course this is a very serious matter when taken in connection with the constant loss due to imperfect insulation. The yield at low pressures would be smaller even than it is, if it were not for the fact that the compressor pumps more air as the pressure falls, as is shown in the third column.[2] It

[1] Quite recently this view has been vigorously assailed by Pictet. (Zeitschrift für comprimirte und flüssige Gase, 7, 1903, 1.)

[2] Due to decreased piston leakage, to the diminishing effect of clearance in the last three cylinders, to the increasing efficiency of the water jackets and intercoolers, and, in very slight degree indeed, to increased speed in the motor, as it responds to the lighter load. This increase of speed only amounts to about 1 per cent.

will be shown below that under otherwise equal conditions, the yield of liquid air increases with the amount of air pumped.

The Capacity of the Liquefier.

Finally, in order to determine the capacity of the liquefier, it was supplied with varying amounts of air, while all other conditions were maintained as nearly constant as possible.

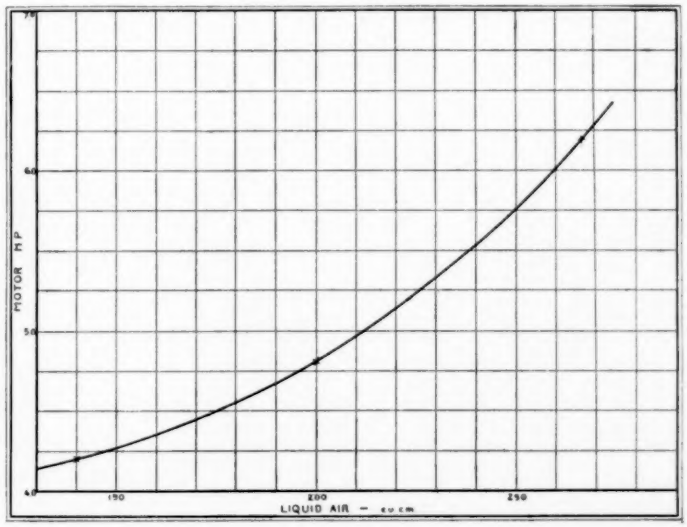
Obviously there are two ways of changing the amount of air delivered by the compressor. One way is to vary the speed at which the compressor is driven. The only practical way of securing this result, except within narrow limits, is to change the size of the driving pulley on the motor. But this involves a separate run for

Table IV.

Effect of Variation in the Air Supply.

Pres.	Air.		H. P.			C.c. Liq. per Eng. H. P.	Per cent. Air Liq.
	Cu. Ft. Per Min.	Liters Liq. Per Hour.	Eng.	Comp.	Liq.		
2,650	13.6	2.66	6.2	4.5	.39	429	8.8
	10.8	2.00	4.8	3.6	.29	416	8.3
	8.1	1.40	4.2	2.7	.20	333	7.8

Fig. 5.



each speed employed, and the conditions of friction, piston leakage, etc., in the motor and compressor are never the same in different runs, as is amply illustrated by the data already given. The other way is to throttle the intake. This can be done to any extent, and as often as may be desired, without any interruption in the operation of the plant, and this method therefore was adopted.

Diagram III. shows the relation between the indicated horse-power of the motor and the yield of liquid air.

One may see from the diagram how very unfavorably the throttling of the intake effects the plant as a whole. This of course was to have been expected. On the other hand, the data given make it clear that the liquefier has not yet been driven either to its full capacity or to its maximum efficiency. In fact the percentage of air liquefied, as given in the last column of the table, gives no indication that the point of maximum efficiency is close at hand. It is our purpose to attach a larger driving pulley to the motor so as to give the compressor a speed of 180 revolutions per minute — which indeed is that for which it was designed — instead of 133 as in all the tests so far. The yield of liquid air will thus be much increased, and it may be confidently expected that the efficiency of the plant also will be improved. The motor especially should develop ten horse-power much more efficiently than it now does six, which is but little more than one third of its normal load.

SUMMARY.

The chief results of this inquiry may be summarized:

1. The average quantity of liquid air per horse-power per hour in normal runs is 439 c.c. The maximum variation from this average is 15 per cent.

2. The average percentage of air liquefied is 8.4, and the maximum variation is only 4¾ per cent.

3. The horse-power equivalent of the liquid air produced averages 6.2 per cent. of the indicated horse-power of the motor. The efficiency of the plant may be considerably improved by increased compressor speed.

4. Liquid air would cease to accumulate in the reservoir of the liquefier if the initial pressure of the air were to fall to about 700 lbs.

5. The importance of precooling is very great indeed. Apparently, in runs of sufficient duration, and with present arrangements, the plant might yield 700 c.c. of liquid air per horse-power per hour, if solid carbon dioxide and ether were used as a cryogen.

WESLEYAN UNIVERSITY, MIDDLETOWN, CONN.,
June 15, 1904.

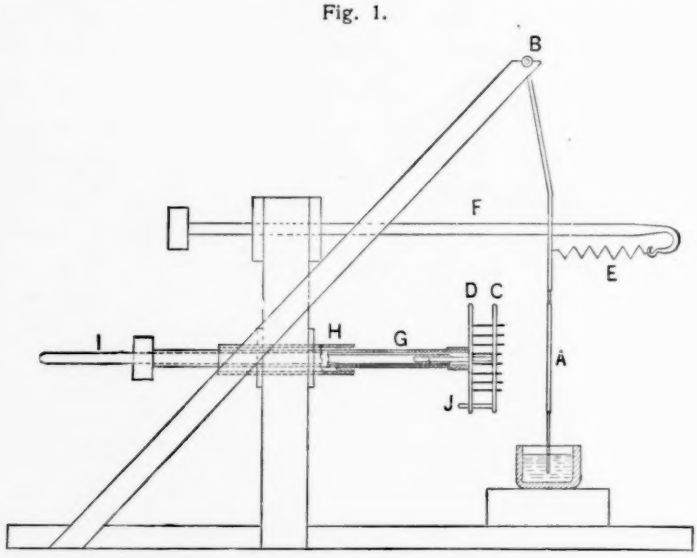
AN AUTOMATIC POTENTIAL REGULATOR.

BY ARTHUR W. GRAY.

AS the writer had occasion to maintain potentials from about 5,000 volts on, which were kept up by a Toepler-Holtz machine driven by an electric motor, more constant than was possible with an ordinary point-overflow to earth, he designed the following simple arrangement, which answered the purpose very well.

A vertical projection of the regulating device is shown in the figure. A light disk *A* of polished metal is suspended by means of

Fig. 1.



an insulating rod from a suitable support so that it can rotate with the least possible friction about a horizontal axis *B*. Opposite *A* is mounted another disk *C*, through small holes in which project a great number of sharp needles fastened into a third disk *D*. Extending downward from *A* is a rod carrying at its lower end a light metallic plate, which serves at the same time to damp the motion of *A* and to make electrical connection between it and some slightly

diluted glycerin contained in a glass trough resting on a block of paraffin. The glycerin has dissolved in it a little calcium chlorid or other salt to render it conducting, and is prevented from creeping over the edge of the trough by slightly greasing the upper part of the glass. A wire dipping into the glycerin connects *A* with the positive terminal of the electrical machine, while another connects *C* and *D* with the negative.

The needle points opposite *A* form a number of ordinary point overflows which are all in operation at the same time; so that the potential difference between *A* and the needles is maintained at a fairly definite value, depending on the distance between them, as long as the source of electricity furnishes a sufficient supply. While the main advantage of this type of regulator over the device ordinarily used consists in the large number of points not too close together, which secures promptness in lessening any increase in the potential due to irregularities in the supply and demand, another gain is effected by the movement of the plate *A*. The attraction due to the electrostatic field between *A* and *C* is counterbalanced by the delicate spiral spring *E*, whose tension is adjusted by pushing the glass rod *F* through the cork that supports it in the framework until *A* is parallel to the plane of the needle points. Any increase in the potential now causes the disk *A* to move towards the points, thus hastening the overflow; while a decrease causes the spring *E* to draw *A* away and lessen the action of the points.

To afford an easy means of adjusting the potential to any desired value, *D* is fastened to a glass tube *G*, which slides with gentle friction through slightly greased corks fixed in the ends of a wider tube *H* fastened to the main framework. By giving the cork handle at the farther end of *G* a slight rotary motion while shoving the tube through *H*, the adjustment may be made almost as delicately as if a screw were used.

With this regulator properly adjusted and connected with a Leyden battery and a good electrical influence machine driven by an electric motor, a potential of twelve thousand volts or more could be maintained for hours within about one per cent. without giving it the slightest attention, although the speed of the driving motor remained by no means so constant; and if the regulation be aided

by slightly moving G with the hand as the electrometer may indicate, the fluctuations may be made much less.

It is not intended that the movement of the disk A should follow sudden fluctuations of the potential due to such causes as erratic breaking down of insulation; its purpose is rather to readjust automatically the distance between the plate and the points in order to accommodate the slower changes that come from slight irregularities in the rate at which the machine keeps up the supply of electricity. To secure the best results, it is necessary to adjust properly the tension of the spring E and also the damping of A. The former should be very delicate and well stretched, that it may contract promptly on the slightest reduction in the strength of the field between A and C. The damping is easily regulated by varying the level of the glycerin. But in addition it is necessary to regulate roughly the distance that the needle points project beyond the plate C. This is done by moving C to or from D by means of the glass tube I, which slides through a cork in the end of G and is fastened to a rod soldered to the back of C and projecting through a hole in D. The short rod J, projecting backward from C through D, is merely to prevent breaking the needles by rotating C while making the adjustment. If the disk C is not far enough behind the points when rather low potentials are employed, A will be set in motion so as to cause rapid fluctuations in the potential; whereas, when high potentials are used, C should be pushed up close to the points so as to increase its influence on A. When A hangs so that it is parallel to the plane of the points, the latter will glow uniformly in the dark. The points should, of course, be sharp and always negative.

BERLIN, PHYSICAL INSTITUTE OF THE UNIVERSITY,
April, 1904

ON THE PRODUCTION OF OZONE IN THE SIEMENS GENERATOR; AND ON AN IMPROVED CONSTRUCTION OF THIS APPARATUS.

By Arthur W. Gray.

1. The writer has described elsewhere[1] some preliminary experiments which he made to compare the quantity of electricity passed through the oxygen in a Siemens ozone generator with the mass of the ozone formed. As was there mentioned, two main sources of error were present, whose effect on the results could not then be estimated, conduction of electricity along the glass surfaces of the generator and failure of the total charge to pass through the galvanometer. In the determinations described in the present paper both of these difficulties were avoided and in addition other improvements conducive to accuracy were introduced. These included a greatly improved key[2] for automatically sending a single charge or discharge through the galvanometer while the generator continued to be charged and discharged in rapid succession, an automatic potential regulator,[3] the use of more concentrated oxygen freed with greater care from water vapor, carbon dioxid, and organic impurities, and the substitution of Schönbein's method of determining the mass of ozone formed by absorbing with potassium iodid and titrating with sodium thiosulfate for Thénard's method of absorbing with sodium arsenit and titrating with iodin.

Greater accuracy was also secured by employing the expression used in calculating the quantity of electricity sent through the gas in a form independent of the shape and dimensions of the generator, so that all the quantities involved could be determined by purely electrical measurements.

In addition to investigating the formation of ozone from concen-

[1] A. W. Gray, Sitzungsber. d. k. Akad. d. Wissensch. zu Berlin, p. 1016, 1903; Ann. d. Phys., 13, p. 477, 1904.

[2] A. W. Gray, Phys. Rev., Vol. XIX., p. 293.

[3] A. W. Gray, Phys. Rev., Vol. XIX., p. 344.

trated and dry oxygen, and the capacity of the generator at various electrical potentials, experiments were made to determine in a general way the influence of water-vapor, air, and the lengthening, the duration of the charging current (*i. e.*, reducing the current strength) by the insertion of a very high resistance in the circuit.

BRIEF OUTLINE OF THE METHOD OF INVESTIGATION.

2. A Toepler-Holtz machine driven by an electric motor served as the source of electricity. One pole was earthed while the other was connected with a large Leyden battery and a Braun electrometer. The potential was maintained uniform by means of a specially designed point overflow to earth. While the outer electrode of the generator always remained earthed, the inner one was connected alternately with the Leyden battery and with the earth. From time to time the quantity of electricity used in charging or discharging the generator was measured. To accomplish this the writer constructed an automatic commutator and galvanometer key. By depressing a lever attached to this a *single* charge was caused to pass from the outer electrode through a ballistic galvanometer instead of direct to earth, while the charging and discharging continued in rapid succession without interruption. By depressing a second lever the current on discharging could be measured at will.

On leaving the generator the ozonized gas passed through a three-way cock lubricated with concentrated sulfuric acid and through either one of two ozone-absorbing solutions into the atmosphere. The first was merely to prevent the ozone from escaping into the room, the second for determining the amount formed during an experiment.

The number of times the electricity passed through the gas was recorded by a specially constructed rotation counter attached to the charging and discharging commutator.

In order to make a determination the electrical machine and the commutator were set in operation, the potential adjusted to the desired value, the current of gas allowed to flow through the first solution, and the counting device set at zero. After a few trial galvanometer deflections showed that the electrical part of the apparatus was working uniformly, the three-way cock was turned

to divert the gas stream into the second solution and the counting device was set in operation. From time to time the quantity of electricity used in charging the generator was measured as described above. After 12,000 to 65,000 passages of the electricity through the gas, the number depending on the potential employed and on the other conditions of the experiment, the three-way cock was turned to its original position, the counting device thrown out of operation, and the mass of ozone produced determined by analyzing the second solution.

Calculation of the Quantity of Electricity Passed Through the Gas.

3. As ordinarily constructed, the effective part of a Siemens ozone generator consists of two coaxial cylindrical shells of glass separated by a layer of the gas to be submitted to the action of the electrical discharge — the whole forming a compound condenser of three layers. As long as the potential difference between the electrodes is insufficient to cause a conduction current through the gas, the electrostatic capacity remains constant; but when the stress becomes sufficient to break down the insulation offered by the gas, the capacity is increased by the effect of the conduction. In his previous paper[1] the writer showed how to calculate the quantity of electricity that flows through the second layer of a condenser composed of three infinitely long coaxial shells, when the potential difference between the outer surfaces is changed from any given value V to any other $V + \Delta V$; the first and third layers being assumed to insulate for all values of V, the second to insulate for low values, but to conduct partially or wholly if V exceeds a certain amount. If we substitute in the expressions there given

$$C_1 \equiv \frac{K_1 l}{2 \log_e r_1/r_0}; \qquad C_2 \equiv \frac{K_2 l}{2 \log_e r_2/r_1}; \qquad C_3 \equiv \frac{K_3 l}{2 \log_e r_3/r_2},$$

at the same time remembering that

$$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C_0}$$

[1] A. W. Gray, Ann. d. Phys., 13, p. 477, 1904.

and

$$\frac{1}{C_1}+\frac{1}{C_3}=\frac{1}{C_\infty},$$

it follows that

$$\frac{1}{C_2}=\frac{1}{C_0}-\frac{1}{C_\infty},$$

or

$$C_2=\frac{C_0C_\infty}{C_\infty-C_0}.$$

Substituting this value in equation (5), p. 480, gives

$$Q_2=\frac{C_\infty(Q-C_0\cdot\Delta V)}{C_\infty-C_0}\equiv\frac{C_\infty}{C_\infty-C_0}(C_v-C_0)\Delta V.$$

Here Q is the quantity of electricity that flows to or from the outer electrodes of the condenser, Q_2 the quantity that flows through the middle layer, C_0 the electrostatic capacity when the middle layer insulates, C_v when it conducts partially, and C_∞ when it conducts completely. In this expression all the quantities occurring can be measured electrically; and since it contains no dimensions, it is not restricted in its application to the case of coaxial cylinders, but applies to a condenser of any form whatever. It is also to be noted that the expression remains the same whether the magnitudes occurring in it are measured in C.G.S. electrostatic units, in C.G.S. electromagnetic units, or in coulombs, farads, and volts.

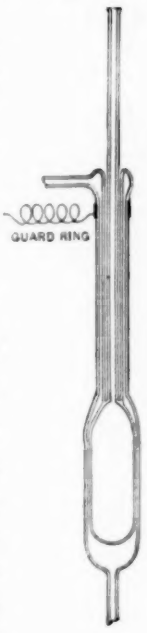
IMPROVED CONSTRUCTION OF THE SIEMENS TYPE OF OZONE GENERATOR.

4. The ozone generator used in the writer's preliminary work was of the usual construction with only a very short length of glass surface separating the walls bounding the gas space, so that it seemed not at all improbable that a portion of the electricity in passing from one side to the other took this path instead of going *through the gas*. As this portion would be measured by the galvanometer, although ineffective in producing ozone, the values obtained for the mass of ozone per coulomb sent through the gas would be too low. A similar error would be caused if the surface separating the electrodes of the generator failed to insulate sufficiently. The

conduction here could be decidedly reduced by coating with fused shellac as was done then while the oxidizing action of the ozone made such an application within entirely out of the question.

To overcome these difficulties the writer constructed the generator represented in longitudinal section by Fig. 1. The effective portion that comes between the electrodes is of large diameter, with thin walls as close together as possible, and short enough to permit a flow of gas sufficiently rapid to prevent the concentration of ozone from becoming so great that the deozonizing action of the electric discharge pointed out by Warburg[1] would have an appreciable effect. The remainder of the generator is of thicker glass. Interior insulation is secured by extending the gas-space above, and by reducing the diameter there; and the flow of gas is facilitated by increasing the distance between the walls in the upper portion as well as by providing a considerable space at the bottom. That the insulation thus obtained was effective, even when considerable water-vapor was present in the oxygen, seems clearly indicated by the results obtained.[2] External surface insulation is increased by carrying the conductor to the inner electrode down through a narrow glass tube sealed into the neck as indicated. As an additional precaution a metallic guard-ring connected to earth surrounded the neck of the generator just below the gas exit. The electrodes were of silver deposited on the glass in order to avoid strains which would be caused by the employment of a liquid.

Fig. 1.



Great care was given to the construction of the generator in order to get the walls bounding the narrow part of the gas space as nearly parallel as possible. The interior was also carefully cleaned by boiling concentrated acids in it for about half an hour. The length of the uniform narrow portion was a little over 5 cm. and the mean radii (determined by weighing with mercury) were: $r_0 = 1.138$ cm.; $r_1 = 1.222$ cm.; $r_2 = 1.284$ cm.; $r_3 = 1.366$ cm. More accurate

[1] E. Warburg, Sitzungsber. d. k. Akad. d. Wissensch. zu Berlin, p. 712, 1900; Ann. d. Phys. 9, p. 788, 1902.

[2] See 12 and 21.

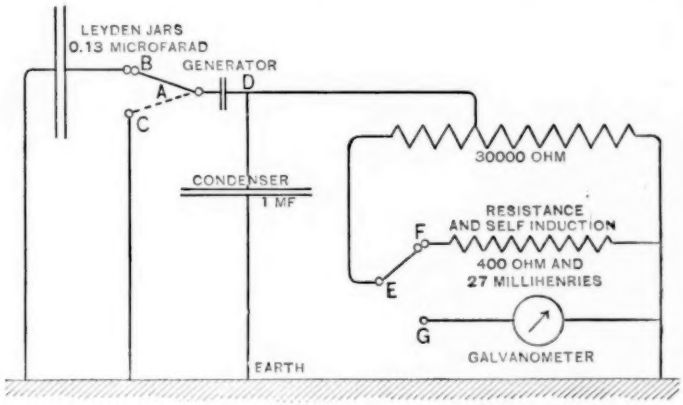
special determinations of the mean thickness of the gas layer gave $r_2 - r_1 = 0.064$ cm. with a probable error of less than 0.001 cm.

ARRANGEMENT OF THE ELECTRICAL CIRCUIT FOR AVOIDING LEAKAGE.

5. Owing to the great self-induction (about 27 millihenries) of the galvanometer employed, which was of the Deprez-d'Arsonval type constructed by Siemens and Halske, considerable difficulty was experienced in getting all of the current to be measured to pass through the entire length of the coil. Attempts were first made in the direction of improving the insulation of the circuit and by inserting a high resistance immediately after the generator, so as to obtain as low a potential as possible in the rest of the path through the galvanometer to earth. But when high potentials were used, the suspended coil behaved as if it were not swinging freely; instead of giving a prompt deflection, it would quiver for a few seconds, and then drift, sometimes slowly, sometimes suddenly, perhaps to the right, perhaps to the left. This behavior had also previously been observed with the similar galvanometer employed in the earlier investigation; but after removing the coil and wiping the pole pieces with the finger, the trouble, for some reason or other, apparently disappeared, so it was assumed that the coil had been catching against a bit of thread or something of the kind; but when the trouble recurred with a different instrument, its cause was searched for until found. Upon removing the brass plate across the front of the magnet and observing the coil while attempting to get a deflection, it was noticed that the coil was violently drawn aside against one of the pole-pieces and held there, while a series of small sparks appeared. Thus the difficulty was evidently due to an electrostatic charge accumulating on the insulating material of the coil on account of the great self-induction. And this effect persisted even when the resistance in series was increased to over forty megohms. Attempts to avoid the self-induction by substituting a Dubois-Rubens iorn-clad galvanometer failed because the brief but strong current changed the controlling magnetic conditions sufficiently to produce a large shifting of the resting-point after every deflection.

The difficulty was, however, finally removed completely by adopting the arrangement indicated in Fig. 2. The essential point was to place a condenser of large capacity in parallel with the galvanometer, so that the current from the generator flowed first into the condenser, which then discharged itself under very low voltage through the galvanometer at a rate governed by the self-induction in the cir-

Fig. 2.



cuit. The motor-driven key *A* connected the inner electrode of the generator alternately through *B* with the Leyden battery, which had a capacity of about 0.13 microfarads, and through *C* with the earth, making about five cycles per second. At the point *D* in the wire leading from the outer electrode of the generator a branch lead to one terminal of a microfarad condenser, whose other terminal was connected with the earth. The wire from the generator also lead to an Ayrton shunt of 30,000 ohms, one end of which was earthed, while the other lead to the key *E*, which was ordinarily connected through *F* with a resistance and self-induction equal to that of the galvanometer (about 400 ohms and 27 millihenries); but which could, by means of an automatic device attached to *A*, be connected during the course of a single charging or discharging with *G*, so that the current would flow through the galvanometer. With this arrangement no resistance was needed in series with the galvanometer.

Since the capacity of the generator was very small compared with the other two, and since the arrangement was in series, the total capacity of the circuit through which the generator was charged,

and also of that through which it was discharged, was a trifle less than its own capacity. Also, the resistance and self-induction of these circuits, being merely that of the connecting wires, was extremely small, affording a very sudden passage of the electricity. To investigate the effect of lengthening the duration of this current (see 10), a resistance of about 42 megohms was inserted between the generator and the point *D*.

The galvanometer was calibrated by substituting for the generator an Elliott standard condenser charged with a single storage cell and removing the microfarad condenser attached at *D*. While the effect of the residual charge introduced a slight error, it was of no special consequence for the present purpose, and would have no effect whatever upon the comparisons of results made later. (9 ff.)

The length of time that the key *A* remained in contact with *B* or with *C* was about 0.047 seconds. Lengthening this by decreasing the speed at which *A* was operated made no perceivable difference in the galvanometer deflection, even when 42 megohms were in the circuit of the generator.

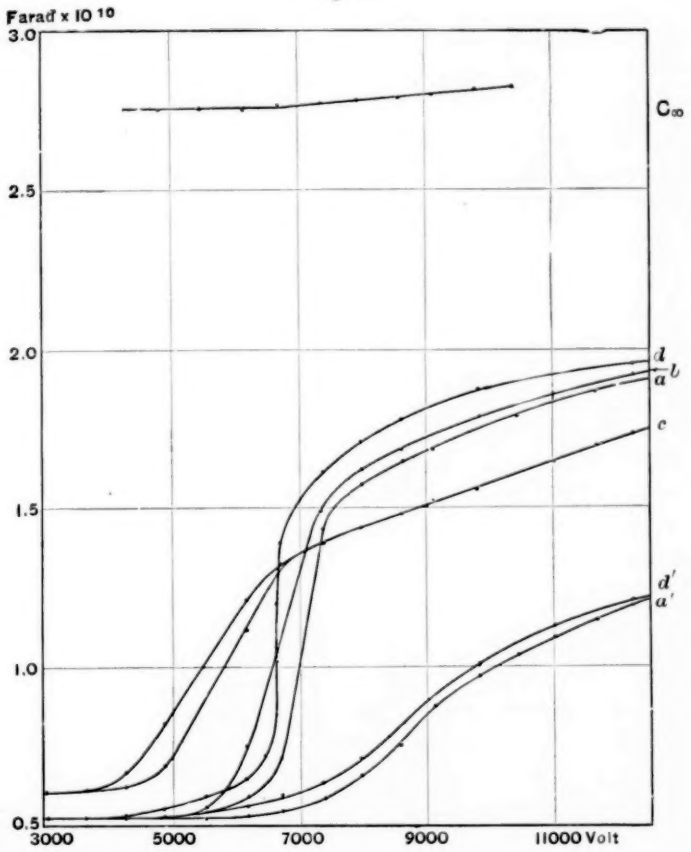
RESULTS.

9. As the writer had observed in his preliminary experiments,[1] a flash of light occurred in the generator when it was charged *suddenly* as well as when it was discharged. Also, with dry oxygen or dry air flowing through it, the same quantity of electricity passed through the circuit on discharging as on charging; but an exception (12) to this was observed when the oxygen contained considerable water-vapor. The results of the measurements, collected in Tables I., II. and III., and represented graphically in Figs. 3, 4 and 5, show, however, not only the unreliability of the previously used method of connecting the galvanometer for the purpose of measuring the quantity of electricity passing through the circuit, but also the serious deficiency of the ordinarily constructed Siemens generator as a means both of carrying out such investigations as are here described, and of applying to the production of ozone the greatest fraction of the electricity employed.

The writer considers all of the determinations described in the present paper as reliable to within about one per cent., while in

[1] A. W. Gray, Ann. d. Phys., 13, p. 486, 1904.

Fig. 3.



many cases greater accuracy was undoubtably reached. Among the thousands of galvanometer deflections that were obtained, not a single unusual one was observed, while in the previous work abnormal ones would appear every now and then.

On account of lack of time for perfecting a means of reliably measuring potentials beyond 12,240 volts, which was the limit of the Braun electrometer employed, it is to be understood that the general statements that follow concerning the description and interpretation of the results are intended to apply only to cases where the potential is less than this value, although some of them, at least, are undoubtedly applicable beyond the range investigated.

THE CAPACITY OF THE GENERATOR AT VARIOUS POTENTIALS.

10. Table I. shows the influence of the potential and other conditions on the electrical capacity of the generator. The first column, V., gives in volts the potential difference applied between the electrodes; the other columns give the corresponding values of the capacity, $C_v \equiv Q/V$, expressed in units equal to 10^{-10} farads. The results recorded under *a* and *a'* were obtained when oxygen, more than 98 per cent. by volume pure, and given its final drying by passing over fresh phosphoric anhydrid, was passed through the generator. Those under *b* illustrate the effect of a very slight amount of water-vapor with the same oxygen, being obtained after the phosphoric anhyrid had been somewhat used up and was coated with a slight liquid film. Renewing the P_2O_5 caused the return of the values under *a*. In obtaining the results under *c* the same oxygen was again used, but the soda lime and phosphoric anhyrid were removed and the gas caused to bubble through distilled water contained in an absorption apparatus of five chambers in series, and then to pass through a considerable length of cotton wool to prevent spray from being carried along into the generator. The temperature of the water was slightly lower than that in the neighborhood of the generator, so the vapor was not quite saturated. The results for air dried with phosphoric anhydrid appear under *d* and *d'*, and those for mercury filling the gas-space under C_∞. While making the measurements for *a*, *b*, *c*, *d* and C_∞, the only resistance in the charging and discharging circuits was that of the conducting wires, whereas while obtaining the last two columns, *a'* and *d'* the duration of the electric current was considerably increased by inserting between the generator and the point *D*, Fig. 2, a resistance of about 42 megohms composed of a solution of cadmium iodid in amyl alcohol. All of these results are represented graphically in Fig. 3, each curve being designated by the letter heading the column from which it was plotted. Except for the low potentials, *a* and *a'* are the mean values taken from Tables II. and III.

During the various experiments in which oxygen was used, the pressure in the generator ranged from 76.2 to 78.4 cm. of mercury, while with air it was about 73.5 cm. A thermometer suspended about four centimeters from the generator indicated a mean tem-

perature during any one series of measurements ranging from 15.6 to 19.7° C. A careful inspection of the results revealed nothing that could be traced to these slight variations of either pressure or temperature.

TABLE I.

V	a	b	c	d	C_∞	a'	d'
3070	0.522		0.609				
3660	0.524		0.615			0.519	
4250	0.525		0.62 0.66	0.53	2.76		0.528
4850	0.523		0.69 0.82	0.55	2.76	0.524	0.533
5500	0.552	0.554	0.91 1.00	0.60	2.76	0.520	0.541
6160	0.588	0.750	1.12 1.21	0.65	2.75	0.530	0.555
6430				0.72			
6600				1.20			
6710	0.717	1.112	1.32	1.40	2.77	0.540	0.600
7380	1.42	1.490	1.39	1.61	2.78	0.587	0.633
7960	1.57	1.637	1.44	1.71	2.78	0.659	0.710
8590	1.64	1.670	1.48	1.78	2.79	0.751	0.801
9220	1.68		1.53		2.80	0.877	
9810	1.76	1.790	1.55	1.87	2.82	0.966	1.002
10410	1.79				2.82	1.034	
11680	1.86		1.69			1.144	
12240	1.90	1.912	1.73	1.95		1.197	1.205

$$C_0 = 0.523. \quad C_\infty = 2.76. \quad Q = \frac{C_\infty}{C_\infty - C_0}(C_v - C_0)V = 1.23(C_v - C_0)V.$$

11. It is to be noticed that none of these curves show the discontinuity observed in the preliminary experiments; but instead, a very rapid rise between certain potentials, provided the electric current is of very short duration, the capacity soon reaching a value between three and four times what it had before the gas began to conduct. When, however, the duration of the current is increased by the insertion of a large resistance, the rise in the capacity begins at a greater potential, takes place much more slowly, and, at least within the range of the potentials here investigated, is very much less, indicating greatly reduced conductivity of the gas, and also that the minimum potential difference between the boundaries of the gas-space, which plays an important part in Warburg's theory of the Siemens generator,[1] depends on conditions other than its construction.

[1] E. Warburg, Verhandl. d. Deutschen Phys. Gesellsch., **22**: p. 382, 1903.

12. A comparison of curves *a* and *b* shows that the presence of a slight amount of water-vapor in the oxygen increases its conductivity a little, but causes the rate of most rapid rise in the capacity to be reduced somewhat. Curve *c*, however, shows some rather remarkable effects of the presence of considerable water-vapor. In column *c* of Table I. it is to be noticed that for values of V between 3,660 and 6,710 volts two values of C_v are given. Those on the left were obtained when the generator was being charged, while those on the right while it was being discharged. Each value given represents a series of consistent galvanometer deflections, and as a check, the same measurements were repeated several hours afterward with the same result of finding that between these potentials a greater deflection was obtained on discharging than on charging; while outside of this region the direction of the current seemed to make no difference.

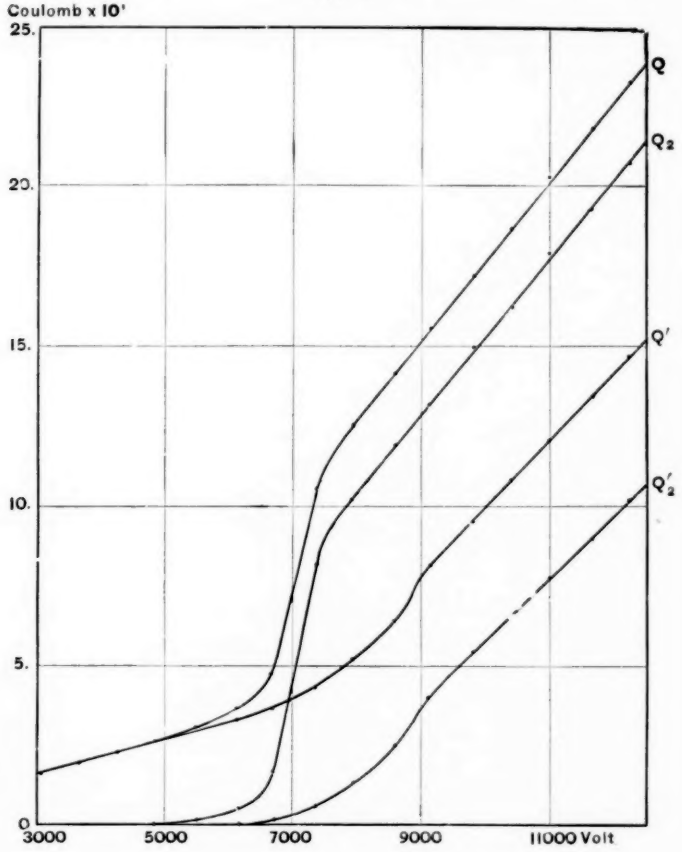
Another point exhibited by curve *c* is that while the presence of considerable water-vapor causes conduction through the gas to take place when the dry gas would insulate, and to reach a much higher value for low potentials; on the other hand, it retards conduction as soon as the potential exceeds a certain value, and from here on the increase in the capacity seems to be directly proportional to the increase in the applied potential. Just what correction may be necessary for the effect of any conduction there may have been along the internal surfaces is not apparent.

It should be remarked in passing that curve *c* is only one of a series of many similar curves representing the conditions for the same generator when water-vapor was present. In a series of twenty consecutive galvanometer deflections, the greatent deviation from the mean would be less than one half per cent.; in fact, the uniformity was greater than with dry oxygen; but an equally consistent series obtained with the same potential several hours later might differ from the first by five per cent. or even more. Just what part was played by differences in humidity, temperature, pressure, or other influencing conditions was not apparent. The results represented in curve *c* were all secured within a short space of time, and represent merely a typical case.

13. Examination of curves *d* and *d'* show that dry air begins to

conduct sooner and conducts better than oxygen; also that the difference between the air and the oxygen becomes less when the electric current is retarded by great resistance or when the potential difference applied between the electrodes of the generator is increased.

Fig. 4.



Air also shows the most rapid rise in the capacity in the neighborhood of the critical potential.

14. When the gas space was filled with mercury, the capacity remained constant up to about 7,000 volts, and then seemed to increase slowly, proportionally to the increase in the potential, at the rate of about $\frac{2}{3}$ per cent. for every additional 1,000 volts. This is apparently due to conduction through the glass.

15. Some interesting information is imparted by Fig. 4, which presents from another point of view the results recorded in columns a (and a') of Table I. The ordinates of curves Q and Q' represent the quantities of electricity used in *charging the generator* at various potentials, those of Q_2 and Q'_2 the quantities that passed as *conduction current through the oxygen.* Q and Q_2 are from the same data as $\bar{a}$ and are taken directly from Table II.; Q' and Q'_2 correspond to $\bar{a}'$ and are from Table III.

Until reaching the critical potential at which conductivity in the gas begins Q and Q' follow the same straight line from the origin of coördinates; similarly Q_2 and Q'_2 both remain zero. In the neighborhood of the critical potential each curve suffers a rapid rise along a curved path and then continues along a straight course which is considerably steeper than before the critical potential was reached; showing that after a certain potential is attained, the increase in the quantity of electricity is directly proportional to the increase in the potential. These lines are all slightly divergent although very nearly parallel. Fig. 4 also makes clear that the lengthening the time required in charging and in discharging the generator reduces not only the *total quantity* of electricity that passes through the circuit, but also the fraction of this (Q_2/Q) that passes as *conduction current through the gas.*

Corresponding curves (not shown here) were plotted for the other cases investigated. All showed the same general characteristics exhibited in Fig. 4. The values of $\frac{dQ}{dV}$ and $\frac{dQ_2}{dV}$ after these curves become straight lines at the higher potentials are given in the following table, where the letters have the same significence as before.

	$\frac{dQ}{dV}$		$\frac{dQ_2}{dV}$	
a.	2.51×10^{-10}		2.47×10^{-10}	coulombs per volt.
b.	2.54	"	1.91	"
c.	2.25	"	1.72	"
d.	2.42	"	1.92	"
a'.	2.11	"	1.89	"

THE PRODUCTION OF OZONE.

16. The results of the measurements made to determine the relation between the mass of ozone produced under given conditions

and the quantities of electricity involved are collected in Tables II. and III. Both series were obtained when commercial oxygen, more than 98 per cent. by volume pure, and passed in succession through a solution of potassium permanganate, glass beads drenched with concentrated sulfuric acid, solid potassium hydroxid, soda lime, phosphoric anhydrid, and cotton wool, flowed through the generator rapidly enough to exclude any possibility of the ozone reaching such a concentration that the deozonizing action of the silent electrical discharge would have an appreciable effect. Nor could any

TABLE II.

V	$C_p \times 10^{10}$.	$C_p - C_0 \times 10^{10}$.	$Q \times 10^7$.	$Q_2 \times 10^7$.	$M \times 10^7$.	M/Q_2.	M/Q.
5,500	0.552	0.029	3.02	0.196			
6,160	0.588	0.065	3.62	0.492			
6,710	0.717	0.194	4.81	1.60			
7,380	1.42	0.901	10.5	8.18			
7,960	1.56	1.04	12.4	10.1	1.47	0.145	0.120
"	1.58	1.06	12.6	10.3	1.43	0.138	0.114
"	1.57	1.05	12.5	10.2	1.45	0.142	0.117
8,590	1.67	1.15	14.3	12.1	1.91	0.157	0.133
"	1.61	1.08	13.8	11.5	1.74	0.152	0.126
"	1.64	1.12	14.1	11.8	1.77	0.150	0.126
"	1.65	1.12	14.1	11.8	1.78	0.151	0.126
"	1.64	1.12	14.1	11.8	1.80	0.152	0.128
9,220	1.67	1.15	15.4	13.0	2.18	0.167	0.142
"	1.69	1.17	15.6	13.2	2.11	0.160	0.136
"	1.68	1.16	15.5	13.1	2.15	0.159	0.139
9,810	1.76	1.24	17.2	14.9	2.64	0.177	0.153
"	1.75	1.23	17.2	14.8	2.58	0.174	0.150
"	1.76	1.23	17.2	14.9	2.61	0.176	0.152
10,410	1.79	1.27	18.7	16.3	2.99	0.184	0.160
"	1.78	1.26	18.6	16.2	2.99	0.185	0.161
"	1.79	1.27	18.6	16.2	2.99	0.184	0.161
11,020	1.85	1.33	20.4	18.0	3.55	0.197	0.174
"	1.83	1.31	20.3	17.8	3.50	0.197	0.173
"	1.84	1.32	20.3	17.9	3.53	0.197	0.173
11,680	1.85	1.33	21.7	19.1	3.90	0.204	0.180
"	1.85	1.33	21.7	19.1	4.06	0.212	0.187
"	1.88	1.36	22.0	19.5	4.03	0.206	0.183
"	1.86	1.34	21.8	19.2	4.00	0.208	0.184
12,240	1.90	1.38	23.3	20.8	4.62	0.222	0.198
"	1.90	1.38	23.2	20.7	4.63	0.233	0.199
"	1.90	1.37	23.2	20.7	4.57	0.221	0.197
"	1.90	1.38	23.2	20.7	4.61	0.225	0.198

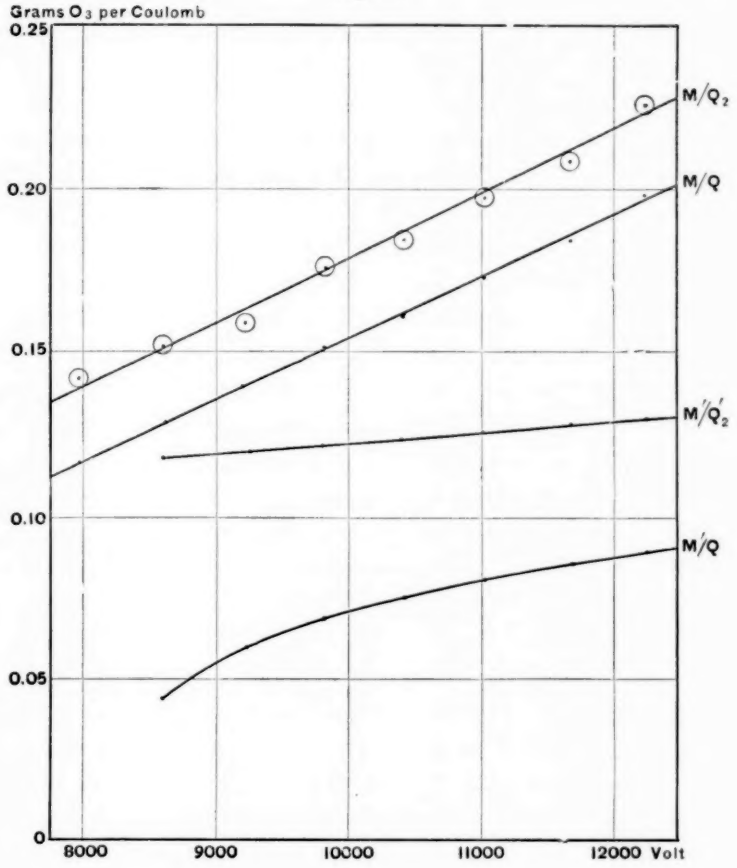
measurable quantity of ozone have escaped absorption; for, while the solution of potassium iodid contained in the first chamber of the absorption apparatus became darkly colored with the liberated iodin, that in the second chamber never exhibited more than a very faint tinge of yellow, and that in the third, fourth, and fifth remained perfectly colorless. In obtaining the results in Table II., the only resistance and self-induction in the charging and discharging circuits was that of the conducting wires, which were for the most part of polished brass about 3 mm. in diameter. Table III. gives the results secured when about 42 megohms were inserted between the generator and the point D, Fig. 2. V, C_v, Q, and Q_2 have the same significance as in 3, and are expressed in volts, farads, and coulombs respectively. The primed letters refer to Table III. M is the number of grams of ozone formed by a single passage of the electricity through the circuit. Just below each single horizontal line are given the means for the same potential of the quantities immediately above.

TABLE III.

V	$C'_v \times 10^{10}$	$\frac{C'_v - C'_6}{\times 10^{10}}$	$Q' \times 10^7$	$Q'_2 \times 10^7$	$M' \times 10^7$	M'/Q'_2	M'/Q'
6,160	0.530	0.007	3.26	0.053			
6,710	0.540	0.017	3.62	0.140			
7,380	0.587	0.064	4.33	0.580			
7,960	0.659	0.136	5.24	1.33			
8,590	0.756	0.233	6.49	2.46	0.292	0.119	0.0449
"	0.746	0.223	6.40	2.35	0.273	0.119	0.0435
"	0.751	0.228	6.45	2.41	0.285	0.119	0.0442
9,220	0.877	0.354	8.07	4.01	0.480	0.120	0.0595
9,810	0.966	0.443	9.47	5.33	0.650	0.122	0.0687
10,410	1.032	0.509	10.74	6.53	0.819	0.125	0.0762
"	1.022	0.499	10.64	6.40	0.778	0.122	0.0732
"	1.037	0.514	10.80	6.59	0.812	0.123	0.0752
"	1.046	0.523	10.89	6.70	0.825	0.123	0.0757
"	1.034	0.511	10.77	6.56	0.808	0.123	0.0751
11,020	1.096	0.573	12.09	7.78	0.981	0.126	0.0812
11,680	1.144	0.621	13.38	8.94	1.140	0.128	0.0854
12,240	1.197	0.674	14.67	10.15	1.296	0.128	0.0885
"	1.198	0.675	14.68	10.16	1.316	0.130	0.0897
"	1.198	0.675	14.68	10.16	1.306	0.129	0.0891

17. Inspection of the columns headed M/Q_2 (represented graphically in Fig. 5) shows beyond question that, contrary to what the

Fig. 5.



preliminary investigation seemed to indicate, the mass of ozone produced per coulomb sent through the gas increases with the potential difference applied between the electrodes of the generator, and that the increase in the former is apparently directly proportional to the increase in the latter. In addition, it is evident that the insertion of the high resistance not only decreases the quantity of electricity that passes as conduction current through the gas, but also decreases a great deal the mass of ozone produced per *coulomb of what is conducted.* And again, the presence of the resistance lessens decidedly the rate at which M/Q_2 increases with the potential. The circles around the points representing the mean values of M/Q_2 are intended

to indicate approximately the degree of reliability that could be reasonably expected. Similar straight lines (not shown here) represent M/Q_2 as a function of Q_2.

18. In Fig. 5 the values of M/Q and M'/Q' are also represented. Here it is to be noticed that, in addition to what is apparent from the discussion of Fig. 4, the mass of ozone produced per coulomb of the *total quantity* of electricity used in charging the generator increases almost, if not quite, in direct proportion to the increase of the potential, provided the resistance in the circuit is negligible; but that this is not at all the case if the resistance is considerable, although proportionality is approached as the potential is increased.

19. Some experiments were made to determine the effect of considerable water-vapor on the production of ozone; but as was mentioned in 12, it was impossible to maintain constant conditions within the generator long enough so that determinations taken several hours apart could be compared with each other. However, in every one of fifteen determinations the yield of ozone per coulomb sent *through the gas* was found to be from ten to fifteen per cent. less than when well dried oxygen was employed with the same potential difference between the electrodes of the generator. And here, too, there *seemed* to be indications that, as long as the gas remained under the same conditions, the increase in the yield of ozone was directly proportional to the increase in the potential.

20. The results described in the last few paragraphs are not out of harmony with those obtained by Warburg [1] in the case of the silent electrical discharge from points. He found that the production of ozone seemed to be closely connected with the amount of illumination, and that when the positive brush discharge made its appearance, the ozone formed per coulomb increased rapidly with the current strength, although the ozone produced by the negative discharge seemed to be independent of the current. In the writer's experiments increase in potential was accompanied by increase in current. In the Siemens generator both positive and negative light are present, and it also seems reasonable to expect that the amount of illumination would be influenced by such things as the resistance

[1] E. Warburg, Sitzungsber. d. k. Akad. de Wissensch. zu Berlin, p. 1011, 1903; Ann. d. Physik., 13, p. 464, 1904.

in the circuit, the nature of the gas, and the presence of water-vapor. The Siemens generator, however, gives a much greater yield of ozone for a given quantity of electricity sent through the gas. It is possible that the great difference in current density in the two cases may have considerable influence.

Comparison with the Results of the Preliminary Experiments.

21. It remains to point out a few things which seem to offer plausible explanations for certain differences between the results here described and those of the preliminary experiments. The discrepancies appear to arise mainly from the two sources of error referred to in 1; and these errors in the earlier work were evidently much greater than was then suspected. But, in addition, inserting a high resistance while using the galvanometer[1] probably changed the conditions within the generator; and it is possible that some differences are to be referred to the reduction of the distance between the walls bounding the gas-space from 0.8 mm. to 0.64 mm.

Poor insulation along the glass surfaces would account for the irregularities and discontinuity previously observed in the neighborhood of the critical potential and would cover up slight differences in conductivity that depended on the condition of the gas used. Surface leakage would also tend to mask the increase in M/Q_2 with the potential. And good evidence of the inadequate internal insulation of the generator employed in the earlier experiments is afforded by some attempts to use moist air and oxygen, which gave values for the capacity only five or ten per cent. less than those obtained when the gas space was filled with mercury.

It is also to be noticed that the greatest value of M/Q_2 that appears in Table II. is considerably less than the smallest value to be found in the corresponding table of the writer's earlier paper. While part of this difference is possibly due to the differences in the generator referred to above, it seems much more reasonable (5) to refer the large values previously obtained to failure of a portion of the electricity to be measured to pass properly through the galvanometer.

[1] A. W. Gray, Physical Review, Vol. XIX., p. 293.

TABLE IV.

V	a	b	c	d	e	f
3,070	1,240		1,190			
3,660	1,480		1,420		1,490	
4,250	1,720		1,640 1,610	1,720		1,720
4,850	1,960		1,820 1,710	1,940	1,960	1,950
5,500	2,200	2,190	1,840 1,750	2,160	2,230	2,210
6,160	2,420	2,240	1,830 1,740	2,360	2,490	2,460
6,430				2,380		
6,600				1,870		
6,710	2,480	2,010	1,750	1,660	2,700	2,630
7,380	1,790	1,700	1,840	1,540	2,900	2,840
7,960	1,720	1,650	1,910	1,520	3,030	2,960
8,590	1,740	1,700	1,950	1,530	3,120	3,050
9,220	1,780		2,060		3,140	
9,810	1,790	1,730	2,150	1,580	3,190	3,120
10,410	1,830				3,250	
11,020	1,840	1,800	2,240		3,320	3,250
11,680	1,910		2,270		3,420	
12,240	1,910	1,890	2,290	1,800	3,470	3,440

APPLICATION OF WARBURG'S THEORY OF THE SIEMENS OZONE GENERATOR.

22. Warburg[1] has given a theory of the Siemens ozone generator in which an important part is played by what he terms the "minimum potential," that is, the potential difference between the boundaries of the gas space when conduction through the gas ceases. Designating this by $\overline{V}_2$ and using the notation employed in the present paper Warburg expresses this in the form

$$\overline{V}_2 = \frac{V}{2}\left[1 - \left(1 - \frac{C_0}{C_2}\right)\frac{C_v}{C_0}\right];$$

this can also be expressed (see p. 350) in the form

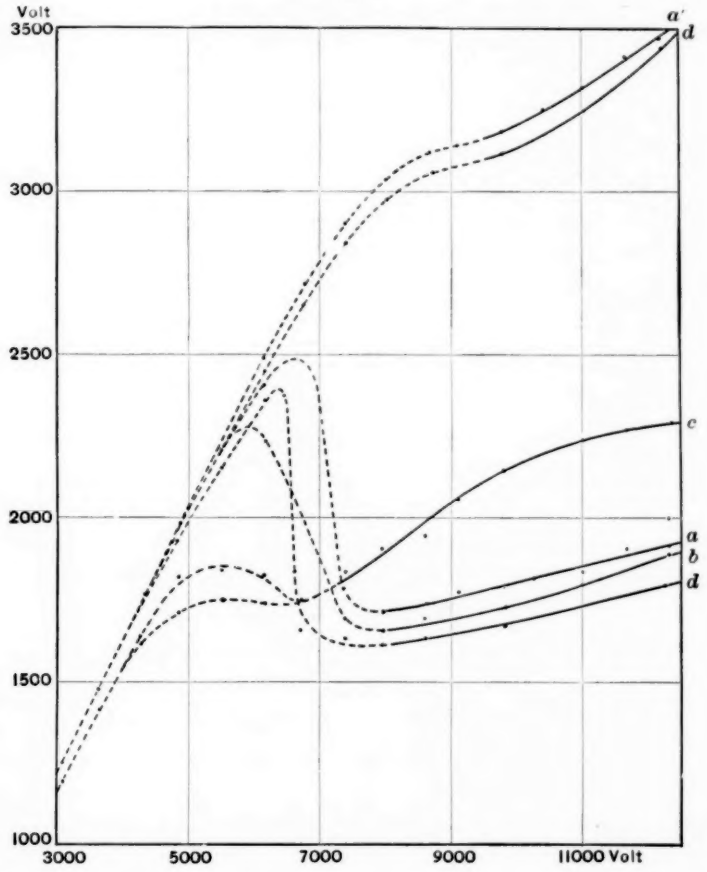
$$\overline{V}_2 = \frac{V}{2}\left(1 - \frac{C_v}{C_\infty}\right).$$

This form is much better adapted to the determination of $\overline{V}_2$ since all the quantities can be easily measured electrically, whereas in calculating C_2 from the dimensions of the generator, very slight errors in measurement have an enormous influence.

[1] E. Warburg, Verhandl. d. Deutschen Phys. Gesellsch. zu Berlin, **22**, p. 382, 1903

From the data in Table I. V_2 has been calculated and recorded under corresponding headings in Table IV. Fig. 6 represents the same facts graphically, the abscissæ being V and the ordinates V_2.

Fig. 6.



The maxima in the curves seem to mark the potential differences between the electrodes at which conduction through the gas begins to take place pretty generally.

Warburg also gives an expression for the potential difference, $(V_2)_0$, between the boundaries of the gas when conduction begins. It is difficult to decide what value of V should be used in this expression to calculate $(V_2)_0$ for a layer of gas 0.064 cm. thick enclosed between glass walls. According to data given by Paschen [1] the

sparking potential difference between metallic spheres 1 cm. in radius and 0.064 cm. apart in ordinary air is about 3,360 volts. This value is obtained for $(V_2)_0$ when a point at about the middle of the steep descent of curve *d* in Fig. 6 is chosen as the basis of the calculation. This corresponds to the point of inflection of curve *d*, Fig. 3.

CONDITIONS FAVORABLE TO THE ECONOMICAL PRODUCTION OF OZONE.

23. From what has been brought out in this paper, it would seem of importance to those engaged in the preparation of ozone for technical purposes to secure the following conditions:

1. Good surface insulation of the generator, both internal and external, by means of some such device as described in 4.

2. Dry oxygen.

3. Charging and discharging as sudden as possible. For this reason the resistance and self-induction of the circuit should be reduced to a minimum, and it is also possible that operation by connection with a Leyden battery somewhat as described might yield better results than the ordinary method of employing an induction coil or an alternating current dynamo, as these machines could not give such a rapid change in the potential difference between the electrodes. Merely placing the generator in parallel with the spark-gap of an electrical machine, as was done by Shenstone and Priest,[2] is inefficient, because the charging takes place so slowly that no illumination,[3] and consequently no ozone,[4] is produced.

In conclusion, the writer wishes to acknowledge his obligations to Professor Warburg for his kindly interest and helpful criticisms, and also to the University of California for generous financial aid extended through the Whiting Fellowship in Physics.

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